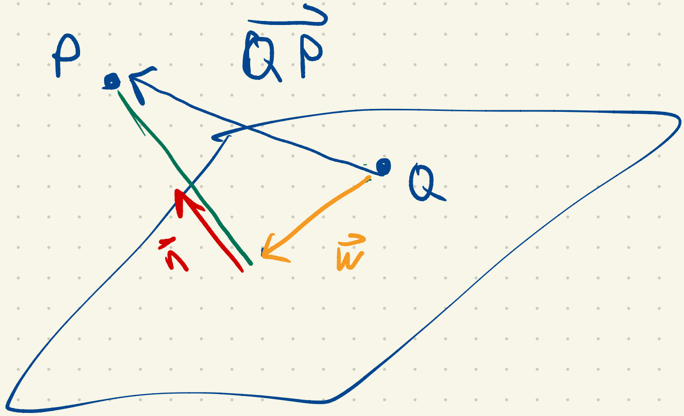
E.g. Distance between P and a plane



$\overrightarrow{QP} = \vec{w} + c\vec{n}$ for some c. ($\vec{w}$ parallel to plane)

distance from P to plane is $\|c\vec{n}\| = |c|\,\|\vec{n}\|$

How to determine c? Take a dot product

$$\overrightarrow{QP} \cdot \vec{n} = \vec{w} \cdot \vec{n} + c\|\vec{n}\|^2$$

$$= c\|\vec{n}\|^2$$

$\overrightarrow{QP} \cdot \vec{n}$

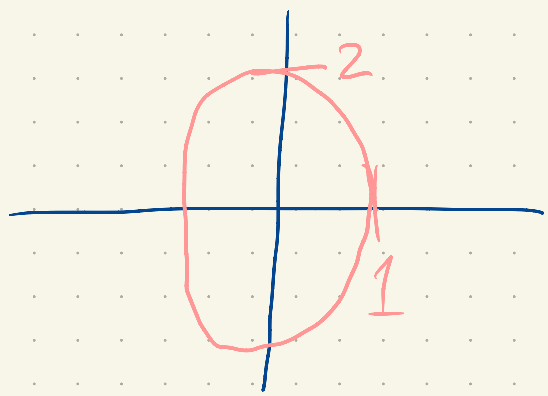
$$|c|\,\|\vec{n}\| = \frac{|\overrightarrow{QP} \cdot \vec{n}|}{\|\vec{n}\|}$$

Surfaces in 3d.

So far we have lines + planes but we'll need other examples

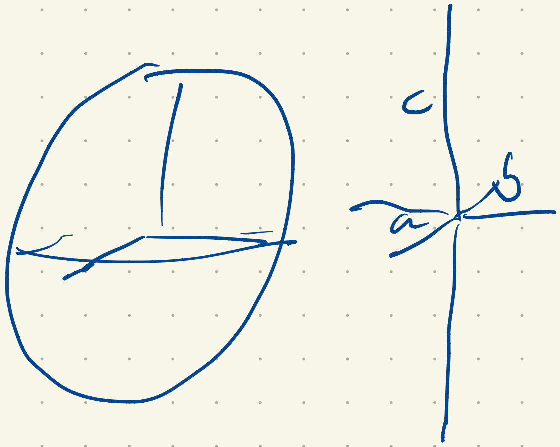
$x^2+y^2=1 \to$ circle of radius 1

$x^2+\frac{y^2}{4}=1$



$x^2+y^2+z^2=1$, r^2 sphere of radius 1, r

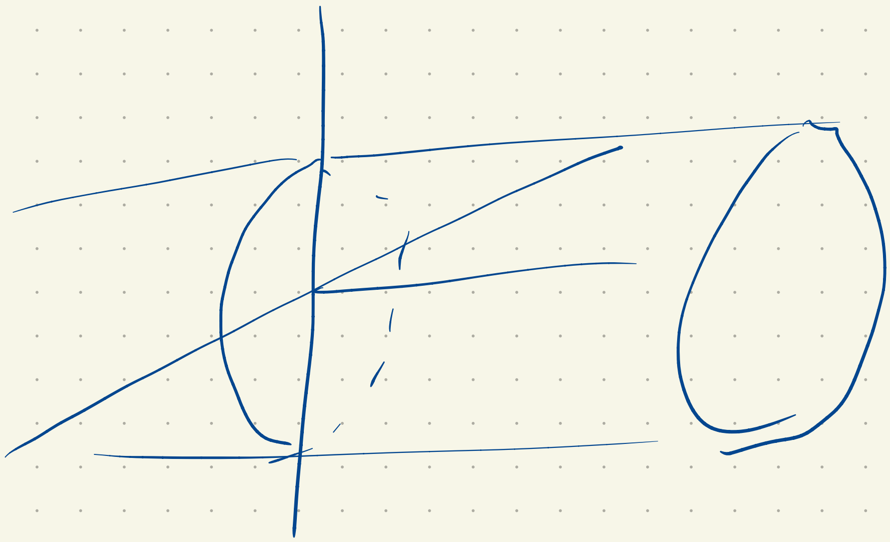
$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2+\left(\frac{z}{c}\right)^2=1$



(Ellipsoid)

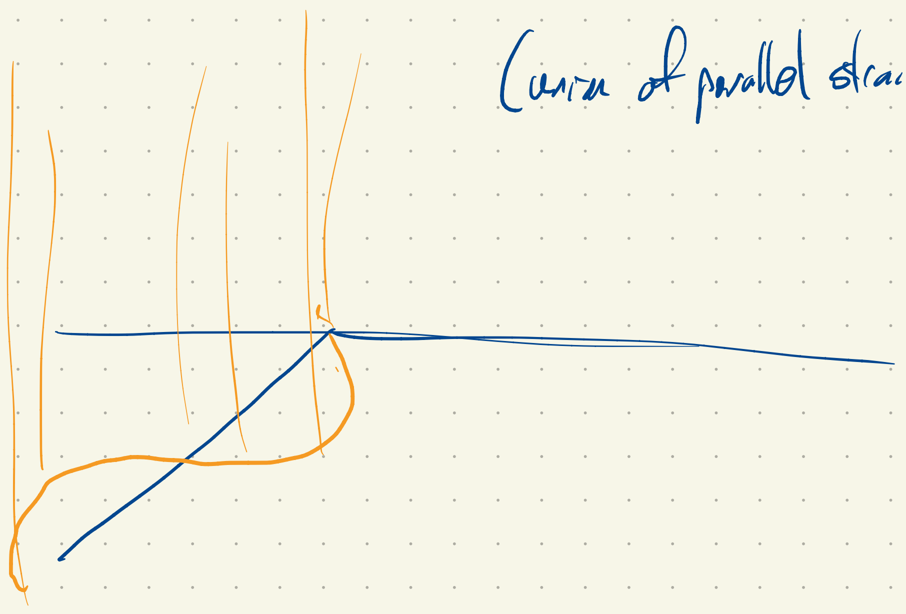
How about $x^2 + z^2 = 1$?

cylinder:



Another: $y = \sin(x)$

(union of parallel straight lines...)



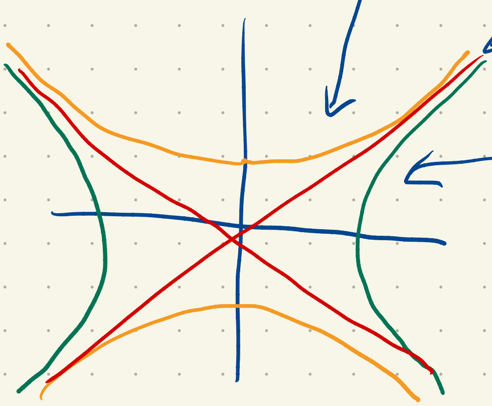
Sec 2.6 Some surfaces in 3-d.

Old friends:

$x^2 + y^2 = 3$ describes circle
center at $(0,0)$
radius $\sqrt{3}$

$y = x^2$ parabola

$y^2 - x^2 = 1$ hyperbola

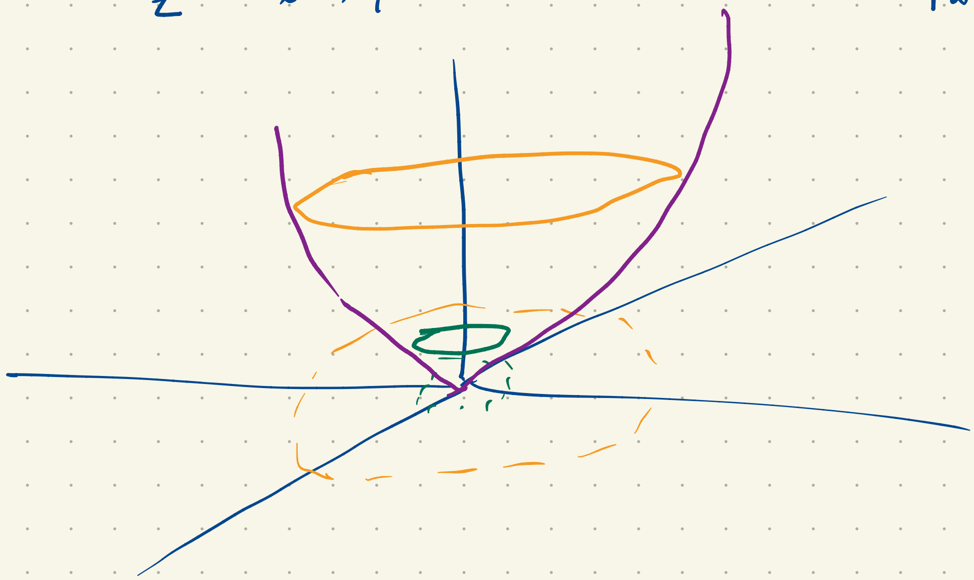


$y^2 - x^2 = 0$
$(y = \pm x)$

$x^2 - y^2 = 1$
$(y^2 - x^2 = -1)$

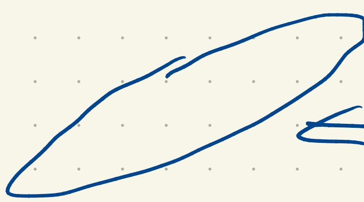
There are generalizations of these in 3-d

$z = x^2 + y^2$ Paraboloid



(So is $x = y^2 + z^2$

$z = \left(\frac{x}{2}\right)^2 + y^2$

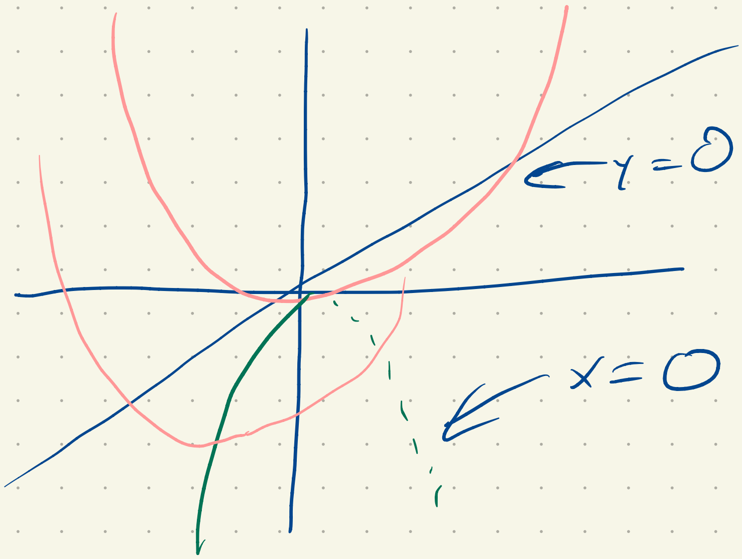


$\left(\frac{x}{2}\right)^2 + y^2 = 1$

(stretch in x - direction!)

My favorites are the hyperbolic paraboloids

$z = x^2 - y^2$



If $y=1$,

$$z = x^2 - 1$$

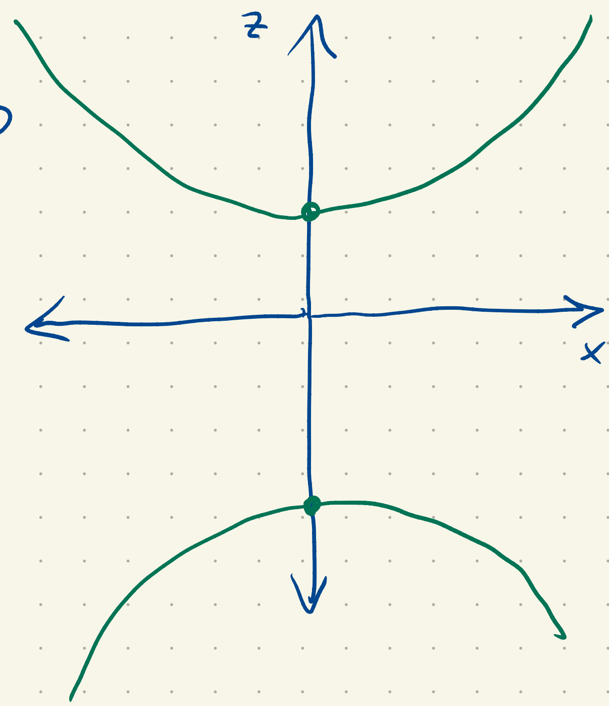
These make saddles.

Cousins: Hyperboloids

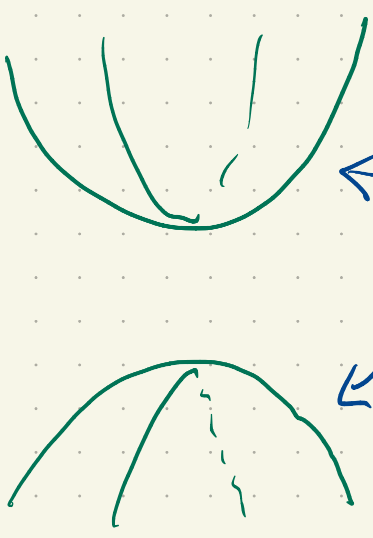
$$z^2 - x^2 - y^2 = 1$$

$z-x$ plane : $y=0$

$$z^2 - x^2 = 1$$



Picture in $z-y$ plane is the same



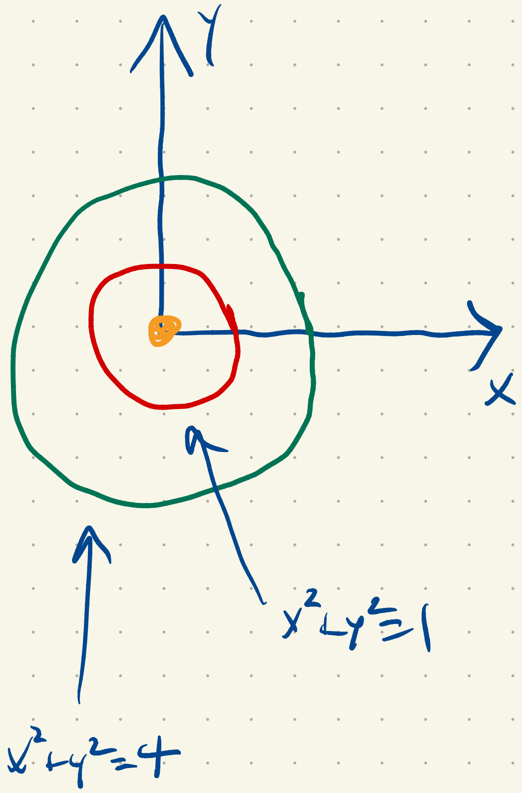
two sheeted hyperboloid

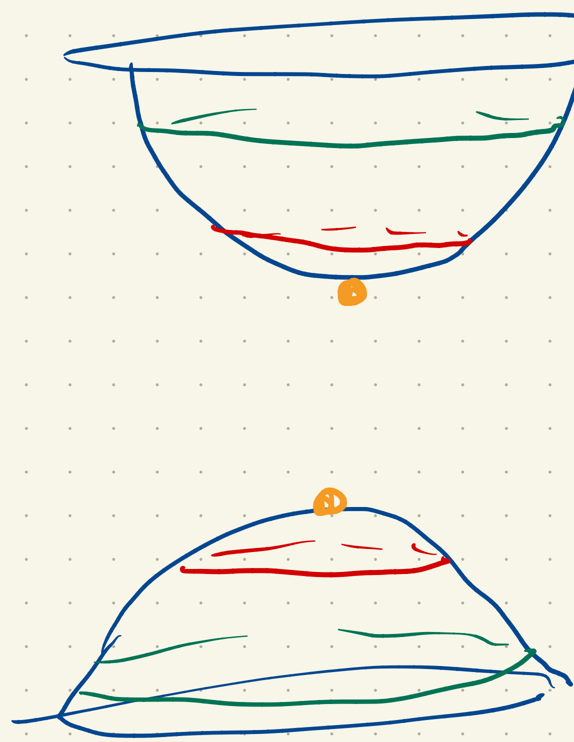
Another way to think about A

$$z^2 - x^2 - y^2 = 1$$

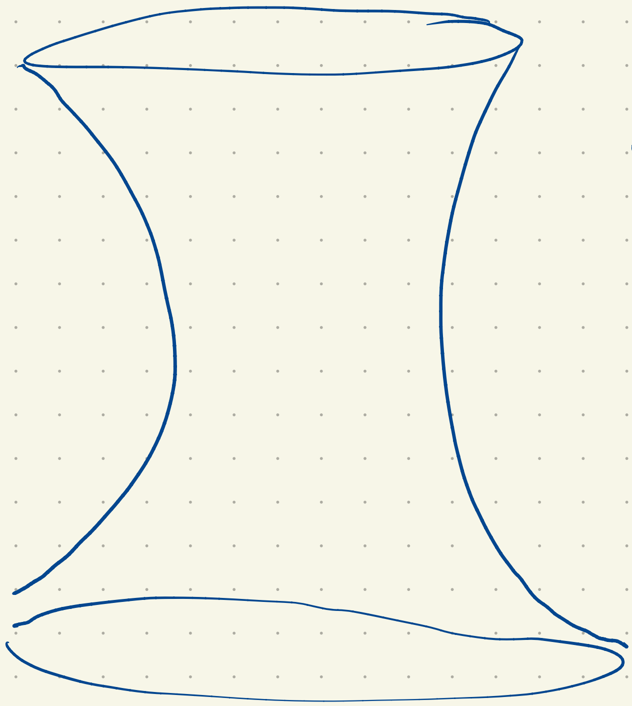
$$z = \pm\sqrt{1 + x^2 + y^2}$$

z coord only depends on $x^2 + y^2$

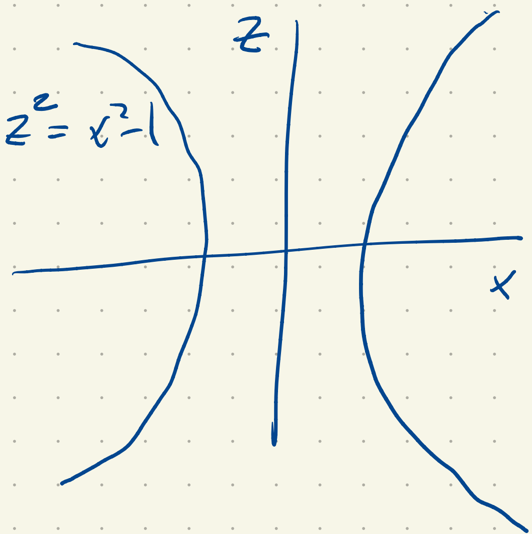




$z^2 = x^2+y^2-1$ ($x^2+y^2<1$ is impossible



one-sheeted hyperboloid

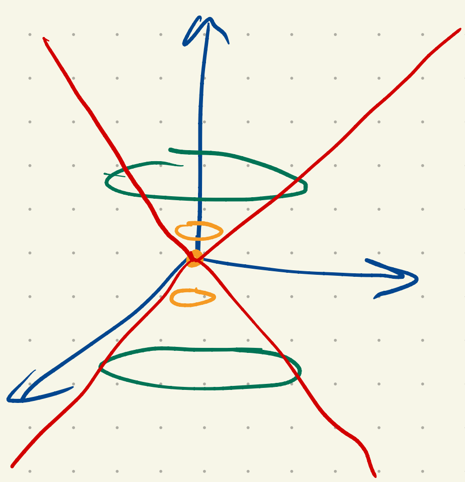


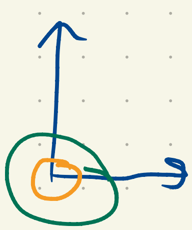
$z^2 = x^2+y^2$

$z^2 = x^2+y^2$ is a cone!

$z^2 = x^2+y^2$

$z = \pm\sqrt{x^2+y^2}$





cone (degenerate hyperboloid)