Lines: You like $y = mx + b$

I like $y = y_0 + m(x - x_0)$

m slope

(x_0, y_0) pt on line

How about 3-d?

3 different forms

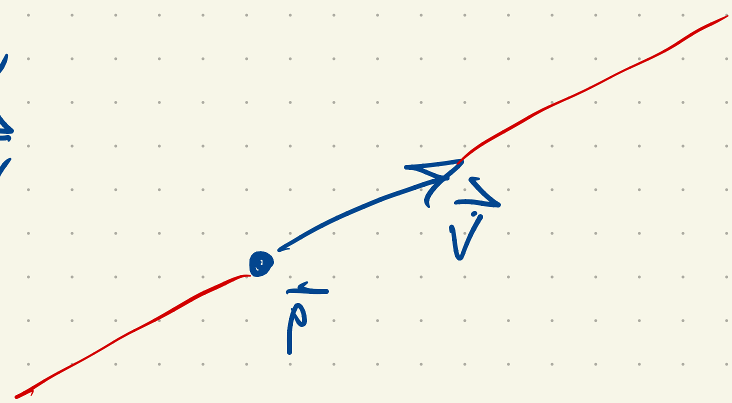
1) Vector form

blurring points vs lines!

Given a point $\vec{p} = \langle x_0, y_0, z_0 \rangle$

and a vector $\vec{v}$,

$\vec{r}(t) = \vec{p} + t\vec{v}$



This form is good for describing all the points, one per choice of t.

E.g. Find line containing $\underset{\vec{p}}{\langle 1, 2, -1 \rangle}$ and $\underset{\vec{v}}{\langle 3, 1, 2 \rangle}$

$$\vec{p} = \langle 1, 2, -1 \rangle$$

$$\vec{v} = \vec{q} - \vec{p} = \langle 2, -1, 3 \rangle$$

$$\vec{r}(t) = \underbrace{\langle 1, 2, 1 \rangle + t\langle 2, -1, 3 \rangle}$$

Vector form

described by a point, and a direction,

$\langle 8, 0, 7 \rangle$

$$\vec{r}(t) = \langle 1+2t, 2-t, 1+3t \rangle$$

$$\left.\begin{aligned} x &= 1+2t \\ y &= 2-t \\ z &= 1+3t \end{aligned}\right]$$

"parametric form"
split into three equations
and essentially the same.

One more: solve for t in above:

$$t = \frac{x-1}{2} \qquad t = \frac{y-2}{-1} \qquad t = \frac{z-1}{3}$$

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-1}{3}$$

This feels wierd. But you can use it to quickly check if a point lies on the line.

$\langle 5, 0, 7 \rangle$? $\quad \frac{5-1}{2} = 2 \quad \frac{0-2}{-1} = 2, \quad \frac{7-1}{3} = 2$ ✓

$\langle 1, 2, 3 \rangle$? $\quad \frac{1-1}{2} = 0 \quad \frac{2-2}{-1} = 0, \quad \frac{3-1}{3} = \frac{2}{3} \neq 0$ X

$\vec{p} = \langle x_0, y_0, z_0 \rangle \quad \vec{v} = \langle a, b, c \rangle$

$$x = x_0 + at$$
$$y = y_0 + bt$$
$$z = z_0 + ct$$

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

You can go back and forth between both forms.

Possible relations: for two lines $\vec{l}_1(t) = \vec{p}_1 + \vec{v}_1 t$

$\vec{l}_2(t) = \vec{p}_2 + \vec{v}_2 t$

1) same!
2) one intersection point

3) parallel

4) none of the above (skew)

1), 3)
v_1, v_2 parallel.
P_1 on ℓ_2 or not

2, 4:
find a pt of intersection or not

See text for examples.

1) and 3) $\vec{d}_1$ and $\vec{d}_2$ are parallel.

1) have all points in common, 3) none

2) and 4) $\vec{d}_1$ and $\vec{d}_2$ not parallel.

solve $\vec{\ell}_1(t) = \vec{\ell}_2(s)$ 3 eq's for s, t.

probably no solution -- (skew!)

Observations

- If you rescale $\vec{v}$, you describe the same line.

- If you change the point to a different point on the same line you describe the same line.

$$l_1 \begin{cases} x = 1+3t \\ y = 2-t \\ z = t \end{cases} \qquad l_2 \begin{cases} x = -2+4s \\ y = 3+s \\ z = 5+2s \end{cases}$$

$$t = 5+2s$$

$$2-(5+2s) = 3+s$$

$$-3-2s = 3+s$$

$$0 = 6+3s$$

$$s = -2 \qquad t = +1$$

But

$x = 4$ vs $x = -10$

So not

Physical way to think about this:

$\vec{r}_0$ starting point at $t=0$

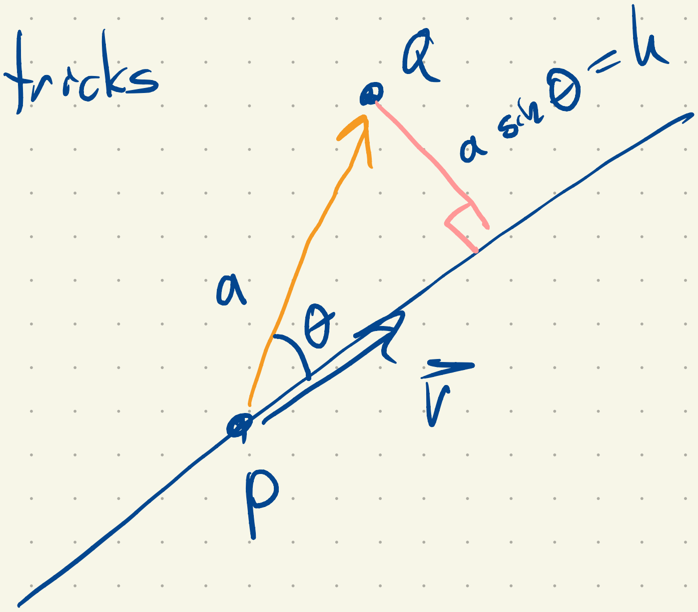
$\vec{v}$ constant velocity.

$\vec{r}(t)$ tells you where an ~~ob~~ object with constant velocity is at each t.

Change $\vec{r}_0$ on the line just changes starting point

Change $\vec{v}$ by scaling just changes speed. (but not the line!)

Fun tricks



How far is $\vec{q}$ from the line? (h)

It's $a \sin\theta$. But $\|\vec{PQ} \times \vec{v}\| = a\|\vec{v}\|\sin\theta$

So $$h = a\sin\theta = \frac{\|\vec{PQ} \times \vec{v}\|}{\|\vec{v}\|}$$

$$= \left\| \vec{PQ} \times \left(\frac{\vec{v}}{\|\vec{v}\|}\right) \right\|$$

unit vector!