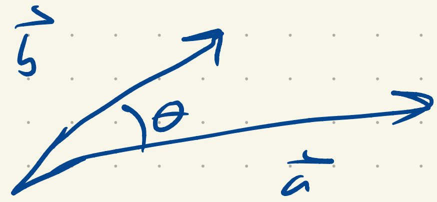
Last class: Introduced cross product

$$\vec{a} \times \vec{b} = \vec{c}$$

vector

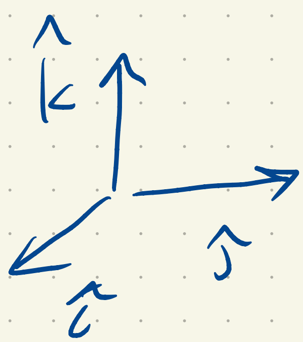


Properties: $\vec{a} \times \vec{b}$ is perp to $\vec{a}$, perp to $\vec{b}$

$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \, \|\vec{b}\| \sin\theta \qquad (0 \le \theta \le 90^\circ)$$

$(\vec{a}, \vec{b}, \vec{c})$ is right handed

(I proved none of this!)



$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

How to compute:

$$\vec{a} \times \vec{b}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

2x2 determinant

$\vec{a} = (a_1, a_2, a_3)$

$\vec{b} = (b_1, b_2, b_3)$

(I'm different from the text!)

$$\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & \boxed{\begin{matrix} a_2 & a_3 \\ b_2 & b_3 \end{matrix}} \\ b_1 & & \end{matrix} \qquad +\hat{i}\,(a_2 b_3 - a_3 b_2)$$

$$\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ \boxed{\begin{matrix} a_1 \\ b_1 \end{matrix}} & \begin{matrix} a_2 \\ b_2 \end{matrix} & \boxed{\begin{matrix} a_3 \\ b_3 \end{matrix}} \end{matrix} \qquad -\hat{j}\,(a_1 b_3 - a_3 b_1)$$

$$\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ \boxed{\begin{matrix} a_1 & a_2 \\ b_1 & b_2 \end{matrix}} & & \begin{matrix} a_3 \\ b_3 \end{matrix} \end{matrix} \qquad +\hat{k}\,(a_1 b_2 - a_2 b_1)$$

$$\vec a \times \vec b = \hat\imath\,(a_2 b_3 - a_3 b_2) - \hat\jmath\,(a_1 b_3 - a_3 b_1) + \hat k\,(a_1 b_2 - a_2 b_1)$$

$$\vec a \cdot (\vec a \times \vec b) = \cancel{a_1 a_2 b_3} - \cancel{a_1 a_3 b_2} - \cancel{a_1 a_2 b_3} + \cancel{a_3 b_1 a_2}$$

$$+ \cancel{a_1 b_2 a_3} - \cancel{b_1 a_2 a_3}$$

$$= 0!$$

$\vec b \times \vec a = -\vec a \times \vec b$ (Check $\vec b \cdot (\vec a \times b) = 0$)
(or use above)

$$\hat\imath \times \hat\jmath \stackrel{?}{=} \hat k$$

$$\begin{matrix} \hat\imath & \hat\jmath & \hat k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$$

$$\hat\imath \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} - \hat\jmath \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} + \hat k \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$0 - 0 + \hat k\,(1^2 - 0)$$

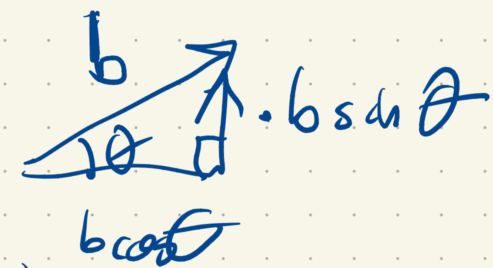
$$= \hat k \checkmark$$

$$\vec{b} = b(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

$\|\vec{b}\|$

$$\vec{a} = a\hat{\imath}$$

$\|\vec{a}\|$



$$\vec{a}\times\vec{b} = a\hat{\imath}\times b(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

$$= ab\cos\theta\,\hat{\imath}\times\hat{\imath} + ab\sin\theta\,\hat{\imath}\times\hat{\jmath}$$

$$\vec{a}\times\vec{a} = \vec{0} \quad \forall \vec{a}$$

$$= ab\sin\theta\,\hat{k} = \underbrace{\|\vec{a}\|\,\|\vec{b}\|\sin\theta}_{\text{length}}\,\hat{k}$$

right handed.

$$\left.\begin{aligned} \vec{a}\times(\vec{b}+\vec{c}) &= \vec{a}\times\vec{b} + \vec{a}\times\vec{c} \\ c(\vec{a}\times\vec{b}) &= (c\vec{a})\times\vec{b} = \vec{a}\times(c\vec{b}) \end{aligned}\right]$$

More properties!

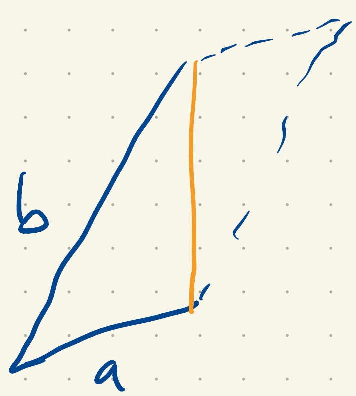
e.g. $\langle 1,2,3\rangle \times \langle 3,1,2\rangle$

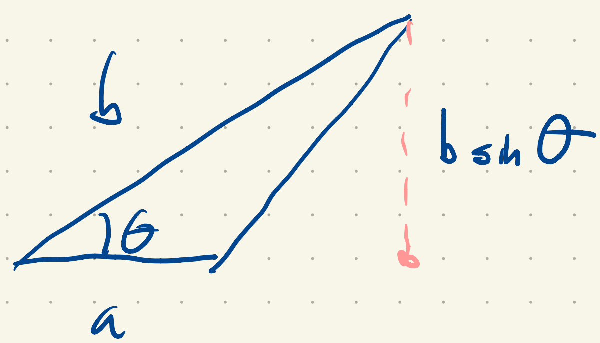
$$\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{array} \qquad \hat{i}\begin{vmatrix}2&3\\1&2\end{vmatrix} - \hat{j}\begin{vmatrix}1&3\\3&2\end{vmatrix} + \hat{k}\begin{vmatrix}1&2\\3&1\end{vmatrix}$$

$$= \hat{i}(4-3) - \hat{j}(2-9) + \hat{k}(1-6)$$

$$= \hat{i} + 7\hat{j} - 5\hat{k}$$

What is this quantity $\|\vec{a}\|\|\vec{b}\|\sin\theta$?





Area of triangle: $\frac{1}{2}ab\sin\theta$

$\|\vec{a}\|\|\vec{b}\|\sin\theta$ is the area of the parallelogram spanned by $\vec{a}$ and $\vec{b}$.