Thu: Suppose X is complete. Then $A \subseteq X$ is complete if ad only if it is closed.

Pf: Suppose A = X is complete, Suppose 2an3
is a sequence in A converges to some
$$x \in X$$
.
We wish to show $x \in A$. Now $2an3 is$
convergent and hence Caudy in X and therefore
Cauchy in A. Since A is complete, the sequence
canveges to some a in A. But converge in
A implies convergence in X. Since limits of sequences
in X are unique, $X = a \in A$.

e.g. R

 $(\mathbb{R}^2, \mathbb{I}_2)$ $\mathbb{I}_1, \mathbb{I}_2, \mathbb{I}_\infty$ l_2 (l_1, l_{ou}, c_0) (CLO, 1], LZ) not complete Given a normed vector space there is a standard tool for showing that it is complete. we say (D) is a backetely conversent if D) is. $\sum_{k=1}^{50} |X_{E}|$ $\sum_{k=1}^{\infty} X_k$ \bigcirc (D)

$$\sum_{k=1}^{60} \frac{c_1k}{k}$$
 is conversed but not absolutely conversed.
From understall analysis: absolutely conversed serves causage.
Thus: A normed linear space X is a Banach space
if all only if every absolutely conversed
serves in the space converses.
Def: A serves $\sum_{k=1}^{60} x_k$ is a booletely conversed if
 $\sum_{k=1}^{60} || x_k ||$ converses.
 $\sum_{k=1}^{60} || x_k ||$ converses.

25: Suppose X is complete and
$$\sum_{k=1}^{\infty} x_k$$
 is
absolutely conversent. We wish to show that the
series converses, i.e. the sequence of partial same converse.
Let $y_{N} = \sum_{k=1}^{\infty} x_k$. Then if $N < M$
 $\| y_{N} - y_{M} \| = \| \sum_{k=N+1}^{M} x_k \| \leq \sum_{k=N+1}^{M} \| x_k \|$.
Since the series $\sum_{k=1}^{\infty} \| x_k \|$ converses, its
service of portial curves is conversent and hence
Cauchy, But then so is $\sum_{k=1}^{\infty} y_{N} = \sum_{k=1}^{\infty} y_{N}$.

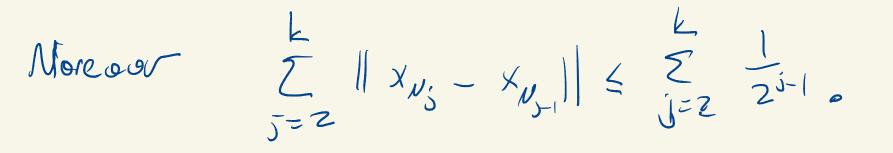
X is complete, le service 2523 conveges, Thtis Zik corvege,

Conversely, suppose absolutely conversent series in X
converse. Let
$$2 \times 3$$
 be a Cauchy sequence in X.
Find N, so that if $n, m = N$, then $|| \times n - \times m|| \leq \frac{1}{2}$.
Find $N_2 > N$ so that if $n, m = N_2$ then $|| \times n - \times m|| \leq \frac{1}{2}$.
(continuing inductively, choose $N_{K+1} > N_k$ such that if
 $N_m = N_k$, $|| \times n - \times m|| \leq \frac{1}{2} \times n$.



Observe

 $X_{N_{k}} = X_{N_{1}} + (Y_{N_{2}} - Y_{N_{1}}) + \cdots + (Y_{N_{k}} - X_{N_{k-1}}).$



By the composition tos, $\sum_{j=2}^{\infty} \|x_{k_j} - x_{j-1}\|$ is conversent

and have $\sum_{j=2}^{\infty} (x_{N_j} - x_{N_{j-1}})$ is conversent also as is But then $\sum_{j=2}^{\infty} x_{N_j}^2$ converse as well.

Coudy sequence in A with a onegent subsequence (MA) and have conveges in A.

Upfalof:

Thus: A subset ASX is compact iff it is complete ad totally bounded.

(X1, Y2, X2, -- ~)

 $Y_{1} = X_{1}$ $Y_{1} + Y_{2} = X_{2}$

Y1442443 = X3

E YK K=1

 $S_{N} = \sum_{k=1}^{N} (Y_{k})$

 $S_N = X_N$

 $\sum_{k=1}^{\infty} \|y_k\| = \sum_{k=1}^{\infty} \|x_k - \dot{x}_{k'}\|$

2 (-1) K-1) K-1 K € (-1) = $\frac{k+l+k}{k(k+l)} = \frac{2k+l}{k^2+l}$ k krl

7 c. K ZYK K+1

7 2 41