1. Carothers 11.65 and this followup:

Show that if $\int_a^b |K(x,t)|\,dt \le 1$ for all $x \in [a,b]$ then the Arzela-Ascoli Theorem implies that given any $f \in C[a,b]$, the sequence $\left(T^{(n)} f\right)_n$ has a subsequence that converges in $C[a,b]$.

2. Suppose $f \in \mathcal{R}[a,b]$ and $\alpha \in \mathbb{R}$. Show that $\alpha f \in \mathcal{R}[a,b]$ and

$$\int_a^b \alpha f = \alpha \int_a^b f.$$

3. Show that the uniform limit of Riemann integrable functions is Riemann integrable. Conclude that $\mathcal{R}[a,b]$ is a closed subspace of $B[a,b]$.

4. Determine if $\chi_\Delta \in \mathcal{R}[0,1]$, where Δ is the Cantor set.