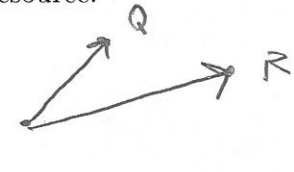
MATH 253 Name: SOLUTIONS

Quiz 2 Due at 11:59 pm in GS on Monday, January 24.

Instructions: (10 pts.) Show all work for credit. You may use your book, but no other resource.

1. (3 pts.) Find the equation of the plane that contains the three points



$$P(1,-1,1), \quad Q(0,4,-1), \quad R(-2,0,1)$$

$\vec{v}_1 = \vec{PQ} = \vec{Q} - \vec{P} = \langle 0,4,-1\rangle - \langle 1,-1,1\rangle = \langle -1,5,-2\rangle$

$\vec{v}_2 = \vec{PR} \quad \vec{R} - \vec{P} = \langle -2,0,1\rangle - \langle 1,-1,1\rangle = \langle -3,1,0\rangle$

} any non-zero scalar multiple will due

One normal is $\vec{n} = -\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -5 & 2 \\ -3 & 1 & 0 \end{vmatrix} = -2\hat{i} - 6\hat{j} + (1-15)\hat{k}$

$= \langle -2,-6,-14\rangle$

To make computation easier, I'll use $n = \frac{-1}{2}\langle -2,-6,-14\rangle = \langle 1,3,7\rangle$.

The equation is $\vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{p}$: $\langle 1,3,7\rangle \cdot \langle x,y,z\rangle = \langle 1,3,7\rangle \cdot \langle 1,-1,1\rangle$

$\Rightarrow x + 3y + 7z = \underbrace{1-3+7}_{5}$

Answer: $\boxed{x+3y+7z=5}$

Your solution might use different vectors, but must be a multiple of my boxed one.

2. (2 pts.) Find the vector projection of the vector $\mathbf{a} = \langle 2,-1\rangle$ onto the vector $\mathbf{b} = \langle 1,3\rangle$.

$\vec{a} = \langle 2,-1\rangle \quad \vec{b} = \langle 1,3\rangle \quad \text{proj}_{\vec{b}}\vec{a} = \underbrace{\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}}_{\text{comp}_{\vec{b}}\vec{a}} \underbrace{\frac{\vec{b}}{|\vec{b}|}}_{\text{unit vector in direction of } \vec{b}}$

Thus, $\vec{a}\cdot\vec{b} = 2-3 = -1$, $|\vec{b}| = \sqrt{1+3^2} = \sqrt{10}$ and $\text{proj}_{\vec{b}}\vec{a} = \frac{-1}{10}\langle 1,3\rangle$

Answer: $\textbf{proj}_{\mathbf{b}}\mathbf{a} = \boxed{\left\langle -\frac{1}{10}, -\frac{3}{10} \right\rangle}$

3. (5 pts.) Consider the two planes given by equations:

Make sure you test that the two normal vectors are not parallel. (Not the equations.)

$$\begin{array}{lrrrl} \text{Plane 1:} & 3x & +y & +2z & =5 \\ \text{Plane 2:} & 6x & +y & +4z & =5 \end{array}$$

(a) (1 pts.) Prove that the two planes are not parallel.

Let $\vec{n}_1 = \langle 3,1,2 \rangle$ and $\vec{n}_2 = \langle 6,1,4 \rangle$ be the normal vectors of the two planes

Since $\vec{n}_1 \neq c\vec{n}_2$ for any scalar c, these normal vectors are <u>NOT</u> parallel.

Thus the two planes are not parallel.

(b) (3 pts.) Since the planes are skew, give the vector and parametric equations of the line of intersection between Plane 1 and Plane 2.

line l of intersection lies in both planes.

Thus, the direction vector $\vec{v}$ for the line l is orthogonal to <u>both</u> $\vec{n}_1$ and $\vec{n}_2$. Take $\vec{v} = \vec{n}_1 \times \vec{n}_2$.

$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 6 & 1 & 4 \end{vmatrix} = (4-2)\hat{i} - (12-12)\hat{j} + (3-6)\hat{k} = \langle 2,0,-3 \rangle$$

To find a point P on the line, I'll notice $P(0,5,0)$ satisfies both plane equations. Thus, l: $\langle 0,5,0 \rangle + t\langle 2,0,-3 \rangle$ $t \in \mathbb{R}$

Lots of correct answers are possible here.

Answer: Vector equation: $\boxed{\langle 2t, 5, -3t \rangle \quad t \in \mathbb{R}}$ Parametric Equations: $\boxed{x(t) = 2t \quad y(t) = 5 \quad z(t) = -3t \quad t \in \mathbb{R}}$

(c) (1 pt.) Prove that the line you found in part (b) is contained in Plane 1.

Check: For all t, $3(2t) + 5 + 2(-3t) \stackrel{?}{=} 5$

$6t + 5 - 6t \stackrel{?}{=} 5$

$5 = 5$ ✓