MATH 253
Quiz 1

Name: _____
Due at 11:59 pm in GS on Tuesday, January 18.

Instructions: (10 points total) Show all work for credit. You may use your book, but no other resource.

1. (2 pts.) A particle is located at the point $P(-1, 5, -10)$, but is constrained so that it can only move in the straight-line direction toward the point $Q(3, 2, -4)$.

   Give, in coordinate form, the position vector $\overrightarrow{PQ}$ representing the direction in which the particle can move.

$\vec{PQ} = \vec{Q} - \vec{P} = \langle 3, 2, -4 \rangle - \langle -1, 5, -10 \rangle = \langle 3-(-1),\ 2-5,\ -4-(-10) \rangle$

$\overrightarrow{PQ} =$ $\langle 4, -3, 6 \rangle$

2. (6 pts.) Consider the two vectors $\mathbf{a} = \langle 2, 2 \rangle$ and $\mathbf{b} = \langle -1 - \sqrt{3}, 1 - \sqrt{3} \rangle$ in $\mathbb{R}^2$.

   (a) Find the unit vector $\mathbf{u}$ pointing in the direction of $\mathbf{a}$.

$\vec{u} = \frac{1}{|\vec{a}|}\vec{a} = \frac{1}{2\sqrt{2}}\langle 2, 2 \rangle = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$ $\quad |\vec{a}| = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$

$= \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

$\mathbf{u} =$ $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$ or better $\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

   (b) Find the angle θ between the vectors $\mathbf{a}$ and $\mathbf{b}$. You final answer should be given as a 'well-known' angle.

If $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$, then $\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}\right)$.

Thus, $|\vec{a}| = 2\sqrt{2}$, $|\vec{b}| = \sqrt{(-1-\sqrt{3})^2 + (1-\sqrt{3})^2} = \sqrt{(1+2\sqrt{3}+3)+(1-2\sqrt{3}+3)}$

$= \sqrt{8} = 2\sqrt{2}$

and $\vec{a} \cdot \vec{b} = \langle 2, 2 \rangle \cdot \langle -1-\sqrt{3}, 1-\sqrt{3} \rangle = -2 - 2\sqrt{3} + 2 - 2\sqrt{3} = -4\sqrt{3}$

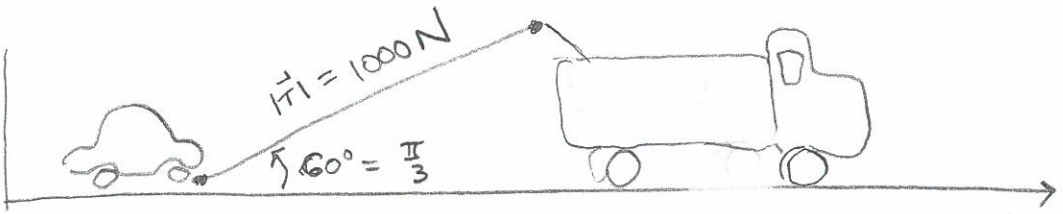
Therefore, $\theta = \cos^{-1}\left(\frac{-4\sqrt{3}}{(2\sqrt{2})(2\sqrt{2})}\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

[QII, ref angle $\frac{\pi}{6}$]

$\theta =$ $\frac{5\pi}{6}$ or 150°

3. (4 pts.) A tow truck drags a stalled car along a level road. The chain makes an angle of 60 degrees with the road and the tension in the chain is 1000 Newtons. How much work is done in dragging the stalled car 2 km? Draw a picture with your solution and include units.

(Your teacher can not draw.)



$\vec{d} = 2000\,\hat{i} = \langle 2000, 0 \rangle$

$\vec{d}$ = displacement vector

Work $= \vec{F} \cdot \vec{d} = |\vec{T}||\vec{d}| \cos\theta$

$= (1000)(2000)\cos\frac{\pi}{3}$

$= 2(1000)^2(\frac{1}{2})$

$= 1{,}000{,}000$

$=$ 1,000,000 Newton-meters

↑ don't forget units