MATH 253 Name: ____________________________

Quiz 11 April 8, 2022

Instructions: (10 points total – 5 pts each) Show all work for credit. You may use your book, but no other resource.

1. In this problem you will show that the line integral

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = 2\pi$$

for the vector field $\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$ and C any positively-oriented, simple, closed circle enclosing the origin. Note that the vector field $\mathbf{F}$ is not defined at the origin, so the domain is the punctured plane.

   (a) Let $C = C_R$ denote the circle of radius R where $R > 0$. Give a parameterization $\mathbf{r}(t)$ for this circle of radius R, where the circle is traversed in the counter-clockwise direction starting and ending at $(R, 0)$.

   Answer: $\mathbf{r}(t) =$ ____________________________

   (b) Using your parameterization, compute the value of the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

2. Consider the two dimensional vector field

$$\mathbf{F}(x,y) = \left\langle e^{xy}(y\sin(x) + \cos(x)),\ xe^{xy}\sin(x) + \frac{1}{y} \right\rangle$$

defined on the upper half plane in $\mathbb{R}^2$ (i.e. $y > 0$)

(a) Prove that $\mathbf{F}$ is conservative on this open, simply-connected domain, then find its potential function $f(x,y)$.

(b) Letting C be the line segment joining $(0,1)$ to the point $(0,\ \frac{\pi}{2})$, compute the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$.