MATH 253 Name: _____

Quiz 3 Due at 11:59 pm in GS on Monday, January 31.

Instructions: (10 pts.) Show all work for credit. You may use your book, but no other resource. Bald answers will be given zero credit.

1. Suppose a particle moves in 3-space and you record its trajectory for times $t \in \left[-\frac{\pi}{8}, \frac{\pi}{8}\right]$. This is given by the space curve with equation

$$\mathbf{r}(t) = \langle \sin(2t),\, \ln\left(\cos\left(2t\right)\right),\, \cos(2t) \rangle \qquad \text{for } t \in \left[-\frac{\pi}{8}, \frac{\pi}{8}\right]$$

where t is measured in seconds and the coordinate functions $x(t)$, $y(t)$, and $z(t)$ in meters.

(a) (2pts.) Give the coordinates of the particle in $\mathbb{R}^3$ at times $t = -\frac{\pi}{8}, 0, \frac{\pi}{8}$. After giving an exact value, round your answer to two decimal places to get a rough estimate for the logarithm.

(b) (2 pts.) By thinking about the coordinate functions, sketch the trajectory of the particle over the time period $-\frac{\pi}{8} \leq t \leq \frac{\pi}{8}$ seconds. Label the three points from part (a) on the trajectory, and put arrows on the path to display the direction of travel. [FYI: The trajectory is not that interesting.] (In part (c), you will add one more vector to this figure.)

(c) (3 pts.) Find the velocity vector $\mathbf{r}'(t)$ and the speed of the particle at time $t = -\frac{\pi}{8}$. Include units in your answer. Finally, returning to (b), draw the velocity vector $\mathbf{r}'(-\frac{\pi}{8})$ with its base (beginning point) at the position of the particle $\mathbf{r}(-\frac{\pi}{8})$.

(d) (3 pts.) Find the distance that the particle travels between time $t = -\frac{\pi}{8}$ and $t = \frac{\pi}{8}$ seconds. Include units and approximate your exact answer to two decimal places.