Name: Solutions

1. A thin wire loop is approximated by the curve C described by $x^2 + y^2 = 3$ with $y \geq 0$; units of length are in cm. The linear density of the wire at a point (x, y) on the loop is given by

$$\rho(x, y) = \frac{1}{4} + \frac{1}{8}y.$$

in grams per centimeter. Compute the mass of the wire by evaluating a line integral with respect to arclength (ds).

$\vec{r}(t) = \langle \sqrt{3}\cos(t), \sqrt{3}\sin(t) \rangle \qquad 0 \leq t \leq \pi$

$\vec{r}\,'(t) = \langle -\sqrt{3}\sin(t), \sqrt{3}\cos(t) \rangle$

$\|\vec{r}\,'\| = \sqrt{3}$

$$\text{mass} = \int_0^{\pi} \left(\frac{1}{4} + \frac{1}{8}\sqrt{3}\sin(t)\right)\sqrt{3}\,dt$$

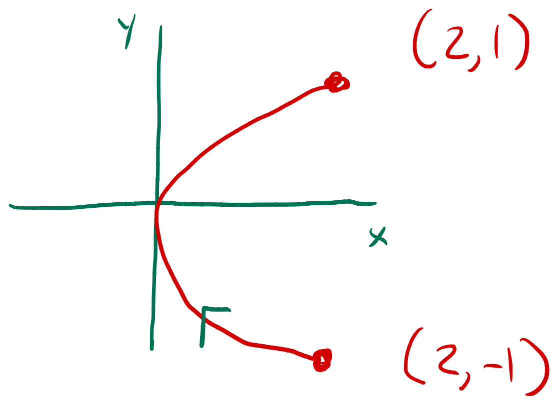
$$= \sqrt{3}\left[\frac{1}{4}t - \frac{\sqrt{3}}{8}\cos(t)\right]\Bigg|_{t=0}^{\pi}$$

$$= \sqrt{3}\left[\frac{\pi}{4} + \frac{\sqrt{3}}{8}\cdot 2\right]$$

$$= \frac{\pi\sqrt{3}}{4} + \frac{3}{4}$$

2. The force $\mathbf{F} = \langle x^2 - y^2, 2xy \rangle$ is (sadly) not conservative. An object experiences this force as it traverses the oriented curve C parameterized by $\mathbf{r}(t) = 2t^2\mathbf{i} + t\mathbf{j}$ for $-1 \le t \le 1$.

1. Sketch the path C of the object in the plane as t increases from -1 to 1. Be sure to add an arrow indicating the direction of traverse. Also, label the coordinates of the endpoints of the curve.



2. Compute the work done by this force on the object as it traverses this curve.

$$\vec{r}\,' = \langle 4t, 1 \rangle$$

$$\vec{F}(\vec{r}(t)) = \langle 4t^4 - t^2, 2 \cdot 2t^2 \cdot t \rangle$$

$$= \langle 4t^4 - t^2, 4t^3 \rangle$$

$$\vec{F}(\vec{r}(t)) \cdot \vec{r}\,' = 16t^5 - 4t^3 + 4t^3 = 16t^5$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_{-1}^{1} 16t^5 \, dt = \frac{16t^6}{6} \Big|_{-1}^{1} = 0$$