Name: Solutions

1. Find all critical points of

$$f(x, y) = 2x^2 + y^4 - 2xy.$$

$$\frac{\partial f}{\partial x} = 4x - 2y$$

$$\frac{\partial f}{\partial y} = 4y^3 - 2x$$

$$\vec{\nabla} f = 0 \Rightarrow \begin{cases} 4x - 2y = 0 \\ 4y^3 - 2x = 0 \end{cases}$$

$\Rightarrow y = 2x$

$\Rightarrow 4y^3 - y = 0$

$\Rightarrow y(4y^2 - 1) = 0$

$\Rightarrow y = 0, \pm\frac{1}{2}$

since $x = y/2$,

$\boxed{\text{crit pts: } (0,0), (\tfrac{1}{4}, \tfrac{1}{2}), (\tfrac{-1}{4}, -\tfrac{1}{2})}$

2. One of the critical points you found should have $y > 0$. Determine if this point is a local minimum, local maximum, or saddle.

$$f_{xx} = 4 \quad f_{xy} = -2 \quad f_{yy} = 12y^2$$

$$D = 4 \cdot 12y^2 - (-2)^2 = 48y^2 - 4.$$

$$y = 1/2 \Rightarrow D = 12 - 4 = 8 > 0$$

$\Rightarrow$ max/min.

Since $f_{xx} > 0 \Rightarrow$ min.

maximize

3. Use the method of Lagrange multipliers to ~~minimize~~

$$f(x, y) = xy$$

subject to the constrant

$$g(x, y) = x + 2y = 5.$$

$\nabla f = \langle y, x \rangle \qquad \nabla g = \langle 1, 2 \rangle$

$\nabla f = \lambda \nabla g \Rightarrow \quad y = \lambda$

$x = 2\lambda.$

Constraint: $x + 2y = 5 \Rightarrow 2\lambda + 2\lambda = 5$

$\Rightarrow \lambda = 5/4$

$\Rightarrow y = 5/4, \; x = 5/2$

max at $(5/2, 5/4)$

max value $5/2 \cdot 5/4 = 25/8$