Name:

**1.** Consider the function

$$f(x,y) = \frac{x^2 - y^3 + xy}{6}.$$

Find the equation to the tangent plane of the graph of $z = f(x,y)$ at $x = 2$ and $y = -1$.

$$\frac{\partial f}{\partial x} = \frac{2x + y}{6} \Rightarrow \left.\frac{\partial f}{\partial x}\right|_{(2,-1)} = \frac{4-1}{6} = \frac{1}{2}$$

$$\frac{\partial f}{\partial y} = \frac{-3y^2 + x}{6} \Rightarrow \left.\frac{\partial f}{\partial x}\right|_{(2,-1)} = \frac{-1}{6}$$

$$f(2,-1) = \frac{4 + 1 - 2}{6} = \frac{1}{2}$$

$$z = f(2,-1) + \left.\frac{\partial f}{\partial x}\right|_{(2,-1)}(x-2) + \left.\frac{\partial f}{\partial y}\right|_{(2,-1)}(y+1)$$

$$= \frac{1}{2} + \frac{1}{2}(x-2) - \frac{1}{6}(y+1)$$

**2.** For the function $f(x,y)$ defined above, it's easy to compute that $f(2,-1) = 1/2$. Use your formula for the tangent plane to estimate $f(2.1,-1.1)$.

$$z = \frac{1}{2} + \frac{1}{2}(x-2) - \frac{1}{6}(y+1)$$

Plug in $x = 2.1$, $y = -1.1$

$$z = \frac{1}{2} + \frac{1}{20} + \frac{1}{60} = \frac{30 + 3 + 1}{60} = \frac{34}{60}$$

**3.** A cylindrical can has volume $V = \pi r^2 h$ where $r$ is the radius of the end and $h$ is the height. Use differentials to esimate the error in the volume of a can if nominally $r = 4$ cm and $h = 10$ cm assuming that both $r$ and $h$ have tolerances of $\pm 0.1$ cm.

$$dV = 2\pi r h\, dr + \pi r^2 dh$$

Substitute $r = 4\text{cm}$, $h = 10\text{cm}$,

$dr = 0.1\text{cm}$, $dh = 0.1\text{cm}$,

$$dV = 2 \cdot \pi \cdot 40 \cdot \frac{1}{10} + \pi\, 16 \cdot \frac{1}{10}$$

$$= \boxed{9.6\pi\ \text{cm}^3}$$