Name: Solutions

1. Consider the function

$$f(x, y) = 10^{-(x^2+y^2)}$$

Sketch the level curves for this function for the values $c = 1$, $c = 1/10$, and $c = 1/100$. Indicate clearly in your diagram which curves correspond to which values of c.

$c = 1 = 10^0 \quad 10^{-(x^2+y^2)} = 10^0$

$-(x^2+y^2) = 0 \quad$ (apply $\log_{10}$!)

$\Rightarrow x = 0$ and $y = 0$

$c = 1/10 = 10^{-1} \quad 10^{-(x^2+y^2)} = 10^{-1} \Rightarrow x^2+y^2 = 1$

$c = 1/100 = 10^{-2} \quad 10^{-(x^2+y^2)} = 10^{-2} \Rightarrow x^2+y^2 = 2$

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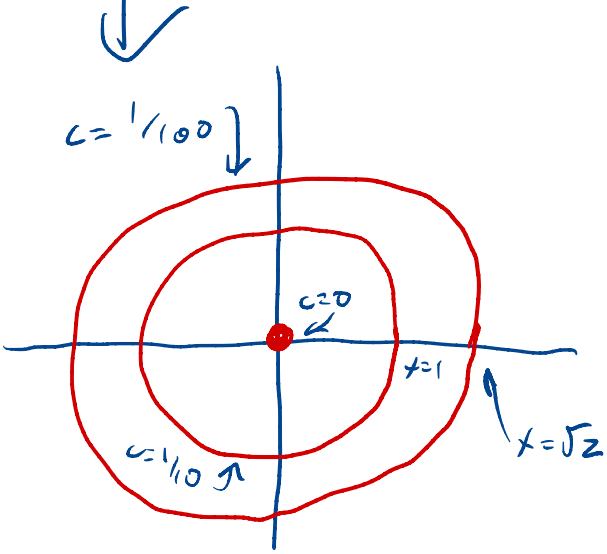
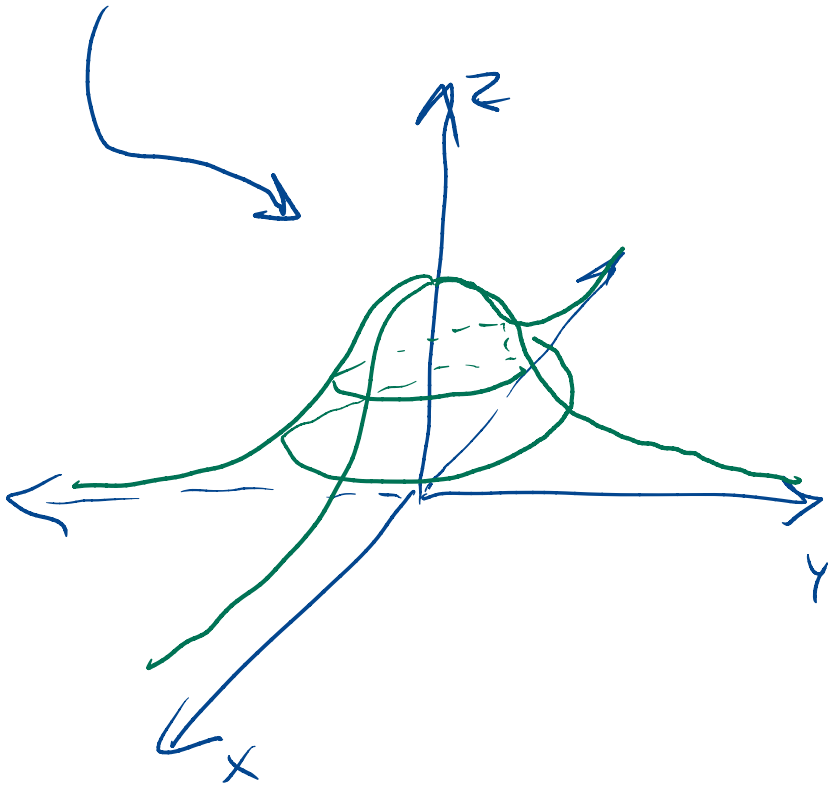
2. Sketch the graph of the function from the previous problem.





3. Consider the function

$$f(x,y) = \frac{xy}{x^2+3y^2}.$$

- Is $(0,0)$ in the domain of this function? Why or why not?

No: division by zero is illegal

- What is the value of this function along the line $y = x$?

$$\lim_{x\to 0} f(x,x) = \lim_{x\to 0} \frac{x^2}{x^2+3x^2} = \lim_{x\to 0} \frac{1}{4} = \frac{1}{4}$$

- What is the value of this function along the line $y = 0$?

$$\lim_{x\to 0} f(x,0) = \lim_{x\to 0} \frac{x\cdot 0}{x^2+3\cdot 0^2} = \lim_{x\to 0} 0 = 0$$

- Either compute $\lim_{(x,y)\to(0,0)} f(x,y)$ or explain clearly why this limit doesn't exist.

The limit does not exist.

The function values approach two different numbers as the inputs tend to the origin along two different lines.