Name:

1. Determine all the **points** of intersection of the parabolic hyperboloid $z = x^2 - y^2$ and the line $\mathbf{r}(t) = \langle 2t, -t, 4t \rangle$.

$$4t = (2t)^2 - (-t)^2$$

$$4t = 3t^2 \implies t(4-3t) = 0$$

$$\implies t = 0,\ 4/3$$

$$\boxed{\begin{aligned} &t=0 && \vec{r}(0) = \langle 0,0,0 \rangle \\ &t = 4/3 && \vec{r}(4/3) = \left\langle \tfrac{8}{3}, -\tfrac{4}{3}, \tfrac{16}{3} \right\rangle \end{aligned}}$$

2. A vector-valued function $\mathbf{r}(t)$ satisfies $\mathbf{r}'(t) = \left\langle e^{2t}, t \right\rangle$. We know additionally that $\mathbf{r}(0) = \langle 1, 2 \rangle$. Determine $\mathbf{r}(t)$.

$$\vec{r}(t) = \int \vec{r}\,'(t)\,dt + \vec{c}$$

$$= \left\langle \tfrac{1}{2}e^{2t}, \tfrac{t^2}{2} \right\rangle + \vec{c}$$

$$\vec{r}(0) = \left\langle \tfrac{1}{2}, 0 \right\rangle + \vec{c}$$

$$\parallel$$

$$\langle 1, 2 \rangle$$

$$\implies \vec{c} = \langle 1/2, 2 \rangle$$

$$\boxed{\vec{r}(t) = \left\langle \tfrac{1}{2} + \tfrac{1}{2}e^{2t}, 2 + \tfrac{t^2}{2} \right\rangle}$$

3. A particle moves on the path

$$\mathbf{r}(t) = \langle 3\sin(2t), 3\cos(2t)\rangle .$$

Show that at each t that $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are perpendicuar.

$$\vec{r}\,'(t) = \langle 6\cos(2t), -6\sin(2t)\rangle$$

$$\vec{r}(t)\cdot\vec{r}\,'(t) =$$

$$3\sin(2t)\cdot 6\cos(2t) + 3\cos(2t)\cdot(-6\sin(2t))$$

$$= 18\,(\sin(2t)\cos(2t) - \cos(2t)\sin(2t))$$

$$= 0$$

$\boxed{\text{Since } \vec{r}(t)\cdot\vec{r}\,'(t) = 0 \text{ for all } t, \text{ these vectors are always perpendicular}}$