Name: **ID:** .

1. Consider the vector $\mathbf{v} = \langle -2, 4, 1 \rangle$. Find its length.

$$\|\vec{v}\| = \sqrt{(-2)^2 + 4^2 + 1^2}$$

$$= \sqrt{4 + 16 + 1}$$

$$= \boxed{\sqrt{21}}$$

2. For the same vector $\mathbf{v}$, find a vector $\mathbf{u}$ pointing in the same direction but that has length 10.

$$\vec{u} = 10 \cdot \frac{\vec{v}}{\|\vec{v}\|} = \frac{10}{\sqrt{21}} \langle -2, 4, 1 \rangle$$

$$= \boxed{\left\langle \frac{-20}{\sqrt{21}}, \frac{40}{\sqrt{21}}, \frac{10}{\sqrt{21}} \right\rangle}$$

3. Find the angle between the vectors $\mathbf{v} = \langle -2, 4, 1\rangle$ and $\mathbf{w} = \langle 1, 1, 0\rangle$. Your answer will use an inverse trig function. That's ok! It might help to notice that the vector $\mathbf{v}$ is the same as in the previous two problems.

$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \, \|\vec{w}\| \cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \, \|\vec{w}\|}$$

$$\|\vec{v}\| = \sqrt{21}, \quad \|\vec{u}\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$$

$$\vec{v} \cdot \vec{u} = -2 \cdot 1 + 4 \cdot 1 + 1 \cdot 0 = 2$$

$$\theta = \arccos\left(\frac{2}{\sqrt{21}\sqrt{2}}\right) = \boxed{\arccos\left(\sqrt{\frac{2}{21}}\right)}$$

4. A large lamp is suspended from the ceiling from two cables and is therefore subjected to three forces: gravitational force $\mathbf{F}_g$ and two tension forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in the cables. The lamp has a mass of 102kg and therefore $\mathbf{F}_g = \langle 0, 0, -1000\rangle$ N. One of the cables provides a tension force $F_1 = \langle 0, -400, 700\rangle$ N. The lamp is in static equilibrium. What is the value of $\mathbf{F}_2$?

$$\vec{F}_g + \vec{F}_1 + \vec{F}_2 = \vec{0}$$

$$\Rightarrow \quad \vec{F}_2 = -\left(\vec{F}_g + \vec{F}_1\right)$$

$$\vec{F}_g + \vec{F}_1 = \langle 0, 0, -1000\rangle + \langle 0, -400, 700\rangle$$

$$= \langle 0, -400, -300\rangle$$

$$\boxed{\vec{F}_2 = \langle 0, 400, 300\rangle}$$