MATH 253
Quiz 2

Name: SOLUTIONS

January 22, 2021

Instructions: Five points total. Show all work for credit. This quiz is closed-book, closed-notes, and no resources at all except for your brain and a pencil and piece of paper. Bald answers will receive no credit. Spend at most 30 minutes on this quiz. Please remember to rationalize denominators and give your answer in 'good' mathematical form.
Gradescope instructions. Scan TWO pages for your solutions.

1. (1 pt.) A snow machine drags a sled along a (flat) trail. The tether makes an angle of 30° with the sled and the tension in the tether is 1200 N. How much work is done in dragging the sled .5 km? Give your answer in joules = Nm.

$\theta = 30$ degrees

$\vec{D}$ $\quad |\vec{D}| = 500m$

$$W = \vec{F} \cdot \vec{D} = |\vec{F}||\vec{D}| \cos 30^\circ$$
$$= 1200 \cdot 500 \cdot \frac{\sqrt{3}}{2}$$
$$= \boxed{300{,}000\sqrt{3} \quad Nm}$$

(or joules)

2. (2 pts.) Give both the parametric equations and the vector equation for the line passing through the point $P(2,1,3)$ and in the direction orthogonal to the plane with equation $x - y + 3z = 1$.

A direction vector $\vec{v} = \langle 1, -1, 3 \rangle$ i.e. the normal $\vec{n} = \langle 1, -1, 3 \rangle$ to the plane

Thus the line has equation $\vec{p} + t\vec{v}$

$$= \langle 2, 1, 3 \rangle + t \langle 1, -1, 3 \rangle$$
$$= \boxed{\langle 2+t, 1-t, 3+3t \rangle \quad t \in \mathbb{R}}$$

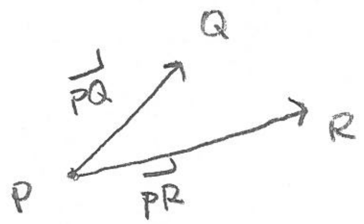
Vector Equation

Parametric Equations:

$x(t) = 2 + t \quad y(t) = 1 - t \quad z(t) = 3 + 3t \quad t \in \mathbb{R}$

3. (2 pts.) Find the equation of the plane that passes through the points

$$P(1,0,3), \qquad Q(2,1,7) \qquad R(3,-1,0)$$



One possibility for the normal vector is $\vec{n} = \vec{PQ} \times \vec{PR}$ (Lots of choices.)

$\vec{PQ} = \langle 2,1,7\rangle - \langle 1,0,3\rangle = \langle 1,1,4\rangle$

$\vec{PR} = \langle 3,-1,0\rangle - \langle 1,0,3\rangle = \langle 2,-1,-3\rangle$

$$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 4 \\ 2 & -1 & -3 \end{vmatrix} = (-3+4)\hat{i} - (-3-8)\hat{j} + (-1-2)\hat{k}$$

$$= \langle 1, 11, -3\rangle$$

with $\vec{n} = \langle 1, +11, -3\rangle$ and point $R(3,-1,0)$,

$$\vec{n}\cdot\vec{r} = \vec{n}\cdot\vec{p}$$

$$\langle 1,11,-3\rangle \cdot \langle x,y,z\rangle = \langle 1,11,-3\rangle \cdot \langle 3,-1,0\rangle$$

$$x + 11y - 3z = 3 - 11 + 0$$

$$\boxed{x + 11y - 3z = -8}$$

Equivalently,

$$-x - 11y + 3z = 8$$