Name:

1. Find all critical points of

$$f(x,y) = 2x^2 + y^4 - 4xy$$

$f_x = 4x - 4y$ $\quad f_x = 0 \Rightarrow x = y$

$f_y = 4y^3 - 4x$ $\quad f_y = 0 \Rightarrow 4y^3 - 4y = 0$

$\Rightarrow y = -1, 0, 1$

Critical points: $(-1,-1)$, $(0,0)$, $(1,1)$

2. You should have found that $(1, 1)$ is a critical point in the previous problem. Classify it as a local minimum, local maximum or saddle point.

$f_{xx} = 4 \quad f_{xy} = -4 \quad f_{yy} = 12y^2$

at $(1,1)$

$D = \begin{vmatrix} 4 & -4 \\ -4 & 12 \end{vmatrix} = 48 - 16 > 0 \Rightarrow$ local min/max (or ambiguous)

$f_{xx} = 4 > 0$

$\boxed{\text{local min}}$

Suppose you wish to maximize $f(x, y) = xe^{-xy}$ subject to the constraint $x^2 + y^2 = 1$. Set up a system of three equations to solve for three variables to find the maximum value. DO NOT ATTEMPT TO SOLVE THE EQUATIONS!

$f_x = e^{-xy} - xye^{-xy}$

$= (1-xy)e^{-xy}$

$g(x,y) = x^2+y^2$

$g_x = 2x$

$g_y = 2y$

$f_y = -x^2e^{-xy}$

equations:

$x^2+y^2=1$ (constraint)

$(1-xy)e^{-xy} = 2\lambda x$

$-x^2e^{-xy} = 2\lambda y$

For unknowns (x, y, λ)