Name:

1. Consider the vector $\mathbf{v} = \langle 1, 2, 1 \rangle$. Find its length and find a unit vector pointing in the same direction as $\mathbf{v}$

$$|\vec{v}| = \left(1^2 + 2^2 + 1^2\right)^{1/2} = \sqrt{6}$$

unit vector: $\left\langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$

2. Find the angle between the vectors $\mathbf{v} = \langle 1, 2, 1 \rangle$ and $\mathbf{w} = \langle 0, 0, -1 \rangle$. Your answer will use an inverse trig function. That's ok!

$$\vec{v} \cdot \vec{w} = |\vec{v}|\,|\vec{w}| \cos\theta$$

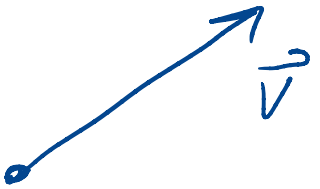
$$\Rightarrow \theta = \arccos\left(\frac{\vec{v} \cdot \vec{w}}{|\vec{v}|\,|\vec{w}|}\right)$$

$$|\vec{v}| = \sqrt{6}, \quad |\vec{w}| = 1, \quad \vec{v} \cdot \vec{w} = -1.$$

$$\theta = \arccos\left(\frac{-1}{\sqrt{6} \cdot 1}\right) = \arccos\left(-\frac{1}{\sqrt{6}}\right) = 1.99\ldots \text{ rad}$$

$$\approx 114.1^\circ$$

3. A steel bar sitting on the ground is pulled by a cable pointing in the (by now familiar) direction $\mathbf{v} = \langle 1, 2, 1 \rangle$ and subjected to a tension force in the cable of 500N. Find the tension force vector $\mathbf{F}_c$ in the cable.



$\vec{F}_c$ is parallel to $\vec{v}$,

$|\vec{F}_c| = 500N.$

unit vector: $\vec{u} = \left\langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$

$$\vec{F}_c = 500N \left\langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle = \left\langle \frac{500}{\sqrt{6}}, \frac{1000}{\sqrt{6}}, \frac{500}{\sqrt{6}} \right\rangle N$$

4. This same steel bar has a mass of 102kg and therefore is subject to a gravitational force $\mathbf{F}_g = \langle 0, 0, -1000\text{N} \rangle$. Find the total force (gravitational and tension) acting on the bar.

$$\vec{F}_c = \left\langle \frac{500}{\sqrt{6}}, \frac{1000}{\sqrt{6}}, \frac{500}{\sqrt{6}} \right\rangle N$$

$$\vec{F}_g = \langle 0, 0, -1000 \rangle N$$

$$\vec{F} = \vec{F}_c + \vec{F}_g = \left\langle \frac{500}{\sqrt{6}}, \frac{1000}{\sqrt{6}}, \frac{500}{\sqrt{6}} - 1000 \right\rangle N$$