Quiz 9
Math 253

Name: Key

11/8

1. Let R be the region inside the unit sphere centered at the origin that also lies in the first octant (where $x, y, z \geq 0$). Assume R has constant mass density k throughout.

   Using spherical coordinates, give an expression involving integrals to compute $\bar{y}$, the y-coordinate of the center of mass of R. **Do not evaluate** any iterated integrals, but be sure they are given in a form where that is all that remains to be done.

$$\bar{y} = \frac{\int\int_0^{\frac{\pi}{2}}\int_0^1 k(\rho\sin\phi\sin\theta)(\rho^2\sin\phi)\,d\rho\,d\phi\,d\theta}{\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^1 k\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta}$$

2. Set up, but **do not evaluate**, an integral that a Calculus II student should understand to find

$$\int_C (x^2 + y)\,ds,$$

   where C is the graph of $y = x^3 + 1$ for $-2 \leq x \leq 3$.

$\vec{r}(t) = \langle t, t^3+1 \rangle \quad t \in [-2, 3]$

$\vec{r}\,'(t) = \langle 1, 3t^2 \rangle$

$ds = \|r'(t)\|\,dt = \sqrt{1+9t^4}\,dt$

So $\int_C (x^2+y)\,ds = \int_{-2}^{3} (t^2+t^3+1)\sqrt{1+9t^4}\,dt$