Quiz 7
Math 253

Name: Key
10/14/19

1. Find all critical points of $f(x,y) = x^3 - 3x + 3xy^2$.

$\nabla f = \langle 3x^2 - 3 + 3y^2, 6xy \rangle = \langle 0, 0 \rangle$

$6xy = 0 \Rightarrow x = 0$ or $y = 0$

If $x = 0$, $3x^2 - 3 + 3y^2 = 0 \Rightarrow 3y^2 = 3 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$

If $y = 0$ $3x^2 - 3 + 3y^2 = 0 \Rightarrow 3x^2 = 3 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

So critical points are

$(0,1), (0,-1), (1,0), (-1,0)$

2. Your answer to part (a) should have included $(1,0)$ as a critical point. What does the Second Derivative Test allow you to conclude about that point?

$$D = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial y \partial x} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 6x & 6y \\ 6y & 6x \end{vmatrix}$$

$$D|_{(1,0)} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix} = 36 > 0$$

$$\left.\frac{\partial^2 f}{\partial x^2}\right|_{(1,0)} = 6 > 0$$

so pt. is a local minimum