

Consider the parameterized lines

$$\begin{aligned}\mathbf{r}(t) &= \langle 1 + 3t, 2 - t, 4 + 6t \rangle \\ \mathbf{s}(t) &= \langle -2 + 5t, 3 + 2t, -2 - t \rangle\end{aligned}$$

1. Show both these lines pass through the point $\langle -2, 3, -2 \rangle$.

$$-2 = 1 + 3t \Rightarrow t = -1$$

$$\vec{r}(-1) = \langle -2, 3, -2 \rangle$$

$$-2 = -2 + 5t \Rightarrow t = 0$$

$$\vec{s}(0) = \langle -2, 3, -2 \rangle$$

2. Since the two lines contain a common point, they lie in a plane. Give a vector orthogonal to this plane.

The direction of \vec{r} is $\vec{a} = \langle 3, -1, 6 \rangle$

The direction of \vec{s} is $\vec{b} = \langle 5, 2, -1 \rangle$

The vector \perp to the plane is

$$\vec{n} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 6 \\ 5 & 2 & -1 \end{vmatrix} = \hat{i}(-1 - 12) - \hat{j}(-3 - 30) + \hat{k}(6 + 5) = \langle -11, 33, 11 \rangle$$

3. Give the equation of the plane containing the two lines.

or $\langle -1, 3, 1 \rangle$

$$\langle -1, 3, 1 \rangle \cdot \langle x, y, z \rangle = \langle -1, 3, 1 \rangle \cdot \langle -2, 3, -2 \rangle$$

$$-x + 3y + z = 9$$