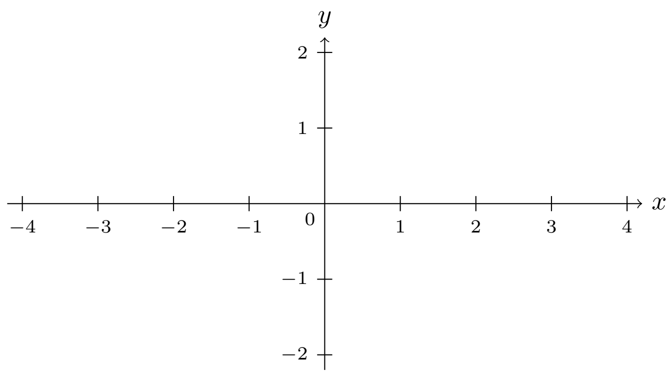
MATH253X-UX1 Midterm Exam 2 Name:________________
Spring 2016

Instructions. (100 points) You have 60 minutes. Closed book, closed notes, no calculator. *Show all your work* in order to receive full credit.

(6pts) **1.** Show that $\lim_{(x,y)\to(2,-1)} \dfrac{xy+2}{x^2+4y}$ does not exist.

(14pts) **2.** Consider the function $z = f(x, y) = x^2 - 4y^2$.

(a) (4 pts) Sketch the level curve $z = 4$.



(b) (8 pts) Use Lagrange multipliers to find the absolute maximum $z_{\max}$ of f on the line $2x + y = 15$.

(c) (2 pts) What is the geometrical relationship between $2x + y = 15$ and the level curves $z = z_{\max}$ at their intersection?

(12pts) **3.** Consider the double integral:

$$I = \iint_R e^{x^2}\, dA$$

where R is the triangular region with vertices $(0,0)$, $(1,1)$, and $(1,-1)$.

(a) (6 pts) Write I as an iterated integral in two ways.

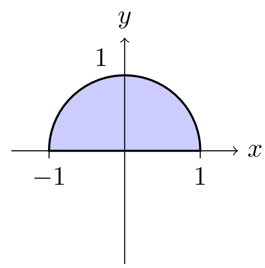
(b) (6 pts) Compute the integral using the form of your choice.

(8pts) **4.** Find an equation of the tangent plane to the surface

$$x^2y - z^2 + \ln(x+y) = 1$$

at the point $(x_0, y_0, z_0) = (-1, 2, 1)$.

(12[pts]) **5.** Compute the mass m of the planar lamina with density $\rho(x, y) = x^2 y$ shown below.

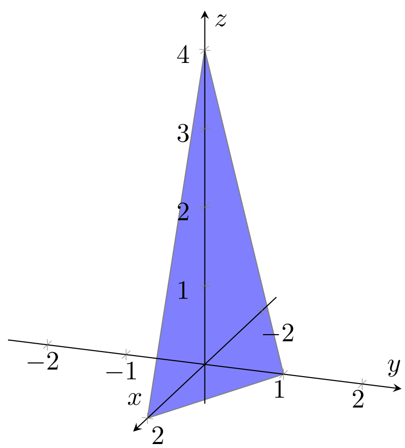


(10[pts]) **6.** Find and classify all critical points of

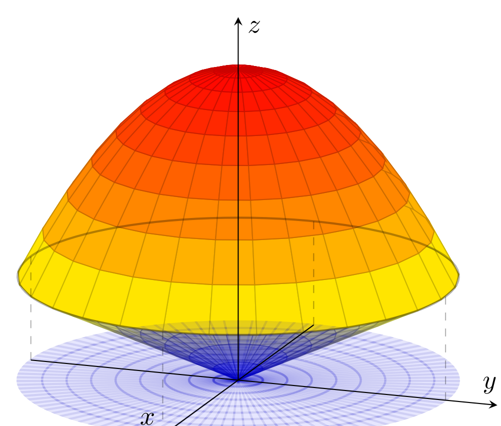
$$f(x, y) = x^2 y - 2x + 4y^2.$$

(20pts) **7.** Fully SET UP bounds and integrands but DO NOT EVALUATE the following double integrals.

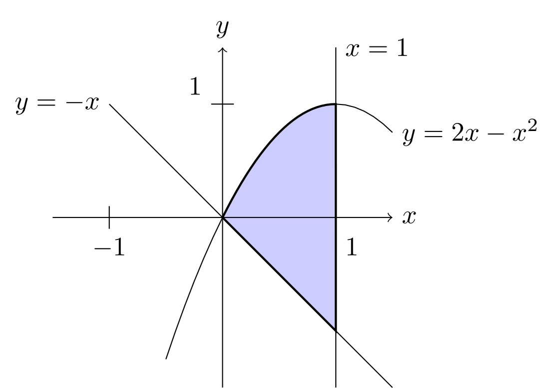
(a) (6 pts) the volume below the plane $2x + 4y + z = 4$ in the first octant:



(b) (6 pts) the volume of the solid bounded by the cone $z = \sqrt{x^2 + y^2}$ and the inverted paraboloid $z = 6 - x^2 - y^2$ using polar coordinates.



(c) (8 pts) the surface area of $z = 4 - x^2 - y$ above the region R bounded by the graphs of $y = -x$, $y = 2x - x^2$, $x = 0$ and $x = 1$ as sketched below:



(18pts) **8.** Let

$$f(x,y) = \frac{x}{x-y}.$$

(a) (6 pts) Compute the maximum rate of change of f at the point $(1,2)$ and specify a unit vector in the direction where this maximum change occurs.

(b) (6 pts) Find the directional derivative of f at $(1,2)$ in the direction of $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$.

(c) (6 pts) Use the differential df to find an approximation of $f(1.1, 1.95)$.