MATH202X-F01/UX1
Spring 2015

Practice Midterm Exam 2

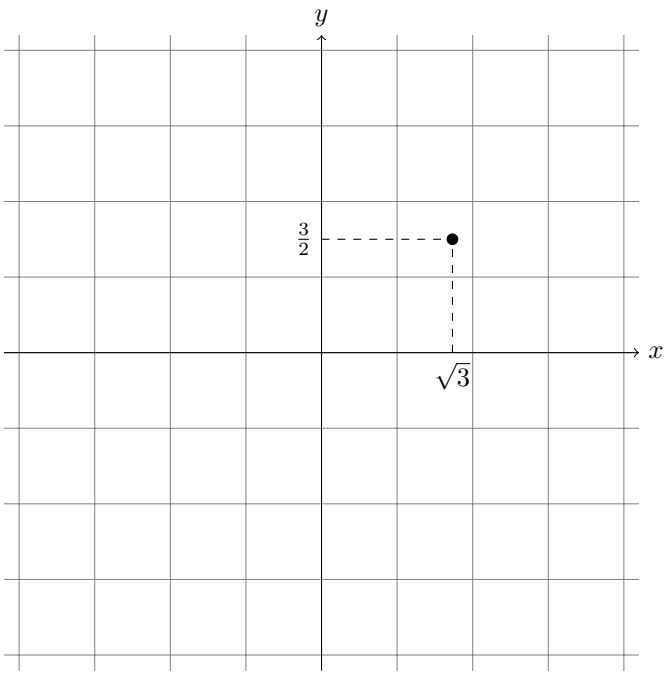
Name: _____

(16^{pts}) **1.**

(a) (8 pts) Prove that $\lim\limits_{(x,y)\to(1,1)} \dfrac{xy-1}{x+y^2-2}$ does not exist.

(b) (8 pts) Consider the surface $z = \dfrac{x^2}{4} + \dfrac{y^2}{9}$ and the point $\left(\sqrt{3}, \frac{3}{2}, 1\right)$ on that surface. Using the axes and grid below,

- Draw the level curve to the surface going through that point.
- Sketch the gradient at that point. Briefly explain your reasoning.



(12^{pts}) **2.** Suppose that $z = x^2 + 2xy - y^2$ where $x = 2u + v$ and $y = u - v$. Find z_u in two ways:

(a) (6 pts) using the multivariable chain rule.

(b) (6 pts) using direct substitution.

(10^{pts}) **3.** Find the equation of the tangent plane to the surface

$$3x\cos y - 2xz^2 + y^2 z = 1$$

at the point $(1, 0, 1)$.

(15^{pts}) **4.** Find and classify the critical points of $z = 2xy - x^2y - \frac{1}{8}y^2$.

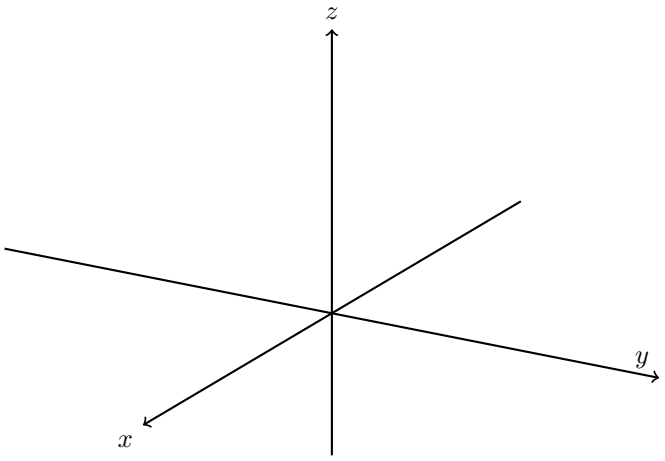
(12$^{\text{pts}}$) **5.** Compute the iterated integral:

$$I = \int_0^e \int_{\ln y}^1 \sin\left(ye^{-x}\right)\, dx\, dy.$$

(12$^{\text{pts}}$) **6.** The total mass of a solid is given by:

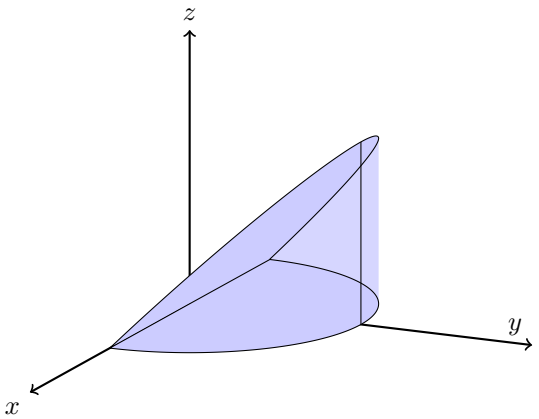
$$m = \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\, dz\, dy\, dx.$$

(a) (6 pts) Describe and sketch the solid in space.

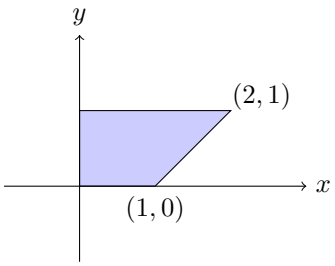


(b) (6 pts) Switch the integral to spherical coordinates. You need not evaluate it but you may choose to do so for extra credit.

(12$^{\text{pts}}$) **7.** Set up a triple integral to calculate the volume of the solid (illustrated below) bounded by the cylinder $x^2 + y^2 = 1$, the plane $y = z$ and the xy-plane. You may use the coordinate system of your choice. Then evaluate the integral.



(11^{pts}) **8.** Consider the following lamina with density $\rho = 2x$.



(a) (6 pts) Find the total mass m of the lamina.

(b) (5 pts) If the first moment about the x-axis is $\frac{17}{12}$, and the first moment about the y-axis is $\frac{5}{2}$, where is the center of mass $(\bar{x}, \bar{y})$ of the lamina?