MATH 253 Name: ______________________________

Midterm 1 February 9, 2021

Instructions: 100 points total. Use only your brain and writing implement. You have 90 minutes to complete this exam. Good luck.

1. (24 pts. – 6 pts. each) A particle at point $P(15, -2, 3)$ in $\mathbb{R}^3$ is constrained so that it can only move along the line segment joining the point P to the point $Q(25, -2, 3)$.

   (a) Find the displacement vector $\overrightarrow{PQ}$ along which the particle may move, and the length $|\overrightarrow{PQ}|$ in meters of $\overrightarrow{PQ}$.

   Answer: $\overrightarrow{PQ} =$ ____________________ $|\overrightarrow{PQ}| =$ ____________________.

   (b) A constant force vector $\mathbf{F} = 2\sqrt{3}\,\mathbf{i} + \mathbf{j} + \sqrt{3}\,\mathbf{k}$ Newtons acts on this particle and moves it from the point P to Q. Find the work done. Include units in your final answer.

   (c) Find the angle θ between the force vector $\mathbf{F}$ and $\overrightarrow{PQ}$.

   Answer: $\theta =$ ______________________________.

   (d) Suppose you wish to **maximize** the work done in moving the particle from P to Q. Find a force vector $\mathbf{G}$ that has the same magnitude as $\mathbf{F}$, but would maximize the workwork done. Briefly justify your answer.

   Answer: $\mathbf{G} =$ ______________________________.

2. (14 pts.) Compute the definite integral

$$\int_0^2 \left\langle 4te^{2t},\ 0,\ \frac{1}{1+4t^2} \right\rangle\, dt.$$

Answer: ____.

3. (12 pts.)

   (a) (9 pts.) Find the equation of the plane containing the points

$$P(-1,2,1), \qquad Q(-1,5,2), \qquad R(-2,1,4)$$

   Answer: ____.

   (b) (3 pts.) Is the origin on this plane? Why, or why not?

4. (24 pts. – 6 pts. each) The formulas for curvature $\kappa(t)$ for a space curve are:

$$\kappa(t) = \frac{\left|\mathbf{r}'(t) \times \mathbf{r}''(t)\right|}{\left|\mathbf{r}'(t)\right|^3} \qquad\qquad \kappa = \frac{d\mathbf{T}}{d\mathbf{s}}$$

Consider the space curve given parametrically by

$$\mathbf{r}(t) = \left\langle 4t,\, t^2,\, \frac{1}{6}t^3 \right\rangle \text{ for } t \in \mathbb{R}.$$

(a) Find the length of the curve $\mathbf{r}(t)$ for $0 \leq t \leq 1$.

Answer: ______________________________.

(b) Find the curvature of $\mathbf{r}(t)$ at the time $t = 1$.

Answer: ______________________________.

(continued ...)

$$\mathbf{r}(t) = \left\langle 4t,\, t^2,\, \frac{1}{6}t^3 \right\rangle \text{ for } t \in \mathbb{R}.$$

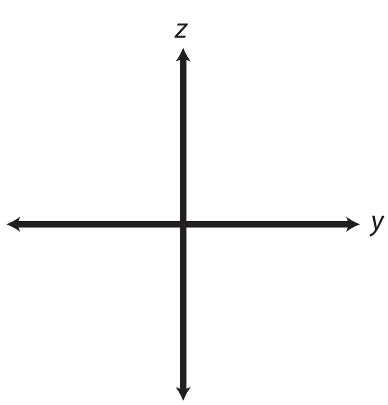
(c) Suppose that a particle's trajectory is given by $\mathbf{r}(t)$ at time t. Give a unit vector $\mathbf{u}$ that points in the direction of travel at time $t = 2$.

Answer: The unit vector is $\mathbf{u} =$______________________________.

(d) Give the parametric equations of the tangent line to $\mathbf{r}(t)$ at the point $\mathbf{r}(2)$.

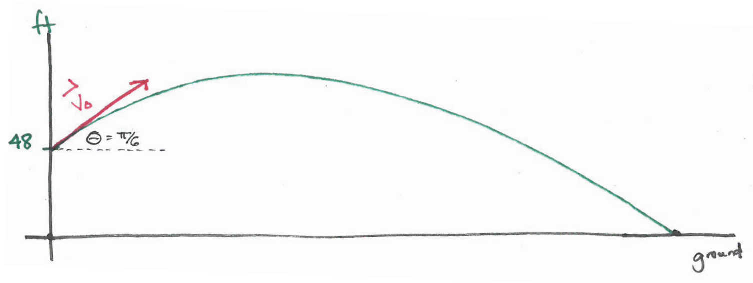
Answer: ____.

5. (9 pts. – 3 pts. each) On the axes below, sketch the x-traces for the values of $k = -9, 0, 1$ for the 'saddle' given by the equation $x = 9y^2 - z^2$. Label the traces with their equations and indicate intercepts as appropriate.



k = -9, 0, 1

6. (17 pts.) A projectile is fired from a height of 48 ft with an initial speed of 64 ft/s, and an angle $\theta = \frac{\pi}{6}$ of elevation. See figure.



(a) (3 pts.) It is not difficult to show that the velecity of the projectile at time t is given by the vector equation:

$$\mathbf{v}(t) = \langle v_x, \, -32t + v_y \rangle \text{ ft/s}$$

where $\mathbf{v}_0 = \langle v_x, \, v_y \rangle$ is the *initial velocity* of the projectile. Find $\mathbf{v}_0$.

Answer: $\mathbf{v_0} =$ ______________________________

(b) (6 pts.) Find the position $\mathbf{r}(t)$ of the projectile at any time t. Include units in your answer.

Answer: $\mathbf{r}(t) =$ ______________________________

(c) (6 pts.) Find the time that the projectile hits the ground, and the horizontal distance it traveled.

Answer: ______________________________

(d) (2 pts.) On the drawing above, sketch an acceleration vector $\vec{a}(t)$ at some time t (you choose) before the projectile hits the ground.