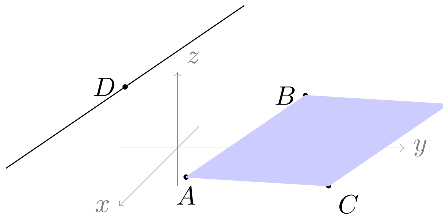
MATH253X-UX1
Summer 2016

Midterm Exam 1

Name: ______________________________

Instructions. (100 points) You have 90 minutes. No calculators allowed. *Show all your work* in order to receive full credit.

(20^{pts}) **1.** Consider the points $A(2,1,0)$, $B(-1,3,1)$, $C(0,4,-1)$, and $D(1,-1,2)$ in space.



(a) (6 pts) Find the symmetric equations of the line going through D and parallel to the line going through A and B.

(b) (8 pts) Find the equation of the plane containing the parallelogram shaded above.

(c) (6 pts) Use vectors to find the length of the diagonal starting at A in the parallelogram.

(15[pts]) **2.** Consider the following planes in space:

$$\begin{array}{ll} \text{Plane 1} & x - 2y - z + 1 = 0 \\ \text{Plane 2} & x - 3y + 2z + 6 = 0 \end{array}$$

(a) (5 pts) Are the two planes orthogonal?

(b) (5 pts) Find the point of intersection of Plane 1 and the line parametrized by

$$\vec{r}(t) = \langle -2 + t, 1 - t, 3 + 2t \rangle .$$

(c) (5 pts) Now find the distance from the point found above to Plane 2.

(10pts) **3.** Let $\mathbf{r}(t) = \left\langle (t-1)^2, t^3 - 3t^2 + 3t, 2t^3 - 3t^2 \right\rangle$ be describing the motion of a particle along a space curve over time. The position is in meters and time in seconds.

(a) (3 pts) Find all the open intervals on which the curve is smooth.

(b) (4 pts) Find the speed of the particle at $t = 2$ s.

(c) (3 pts) Given $\mathbf{s}(2) = \langle 2, 3, -1\rangle$ and $\mathbf{s}'(2) = \langle 1, -1, 2\rangle$, find:

1. $\left.\dfrac{d}{dt}(\mathbf{r} \cdot \mathbf{s})\right|_{t=2} =$

2. $\left.\dfrac{d}{dt}(\mathbf{r} \times \mathbf{s})\right|_{t=2} =$

(8pts) **4.** Time to sketch some surfaces!

(a) (4 pts) Describe and sketch the surface in space whose equation is $x^2 - \dfrac{y^2}{4} + \dfrac{z^2}{9} = -1$.

(b) (4 pts) Describe and sketch the surface in space whose equation is: $x^2 + 4y^2 = z$.

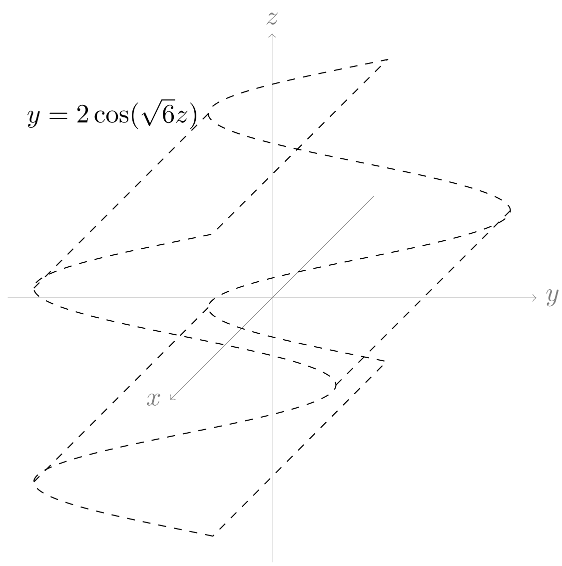
(7pts) **5.** You hit a golf ball in "Calculus III conditions"[1] such that it takes off at an angle of 30° with the horizontal. What should the initial golf ball speed be in order for you to hit a hole-in-one located 800 feet away at the same elevation?

[1] I.e. the acceleration is constant and only due to gravity. That is we ignore ball spin, air resistance, etc.

(15pts) **6.** An object moves along a trajectory so that its position $\vec{r}(t)$ as a function of time is given by:

$$\vec{r}(t) = \left\langle 2\sin(\sqrt{6}t), 2\cos(\sqrt{6}t), t \right\rangle.$$

(a) (5 pts) The particle's trajectory sits on the intersection of the cylinder $y = 2\cos(\sqrt{6}z)$ (drawn below) and which other surface? Sketch that surface. (*Hint:* Even though there is more than one possible answer, one is definitely easier to sketch.)



(b) (5 pts)Find the unit tangent vector of the trajectory.

(c) (5 pts)Find the principal unit normal vector of the trajectory.

(15pts) **7.** A particle is moving in the plane from a starting position at $(-1, 2)$ (i.e. $\vec{r}(0) = -\vec{\imath} + 2\vec{\jmath}$) according to the following ***velocity*** (measured in ft/s) at time t:

$$\vec{v}(t) = (3 - 3t^2)\vec{\imath} + 6t\vec{\jmath}.$$

(a) (6 pts) What is the particle's position at $t = 1$ s?

(b) (9 pts) Find the arc length described by the particle between $t = 0$ s and $t = 2$ s.

(10pts) **8.** Nanook, your favorite sled dog is in training for pulling a loaded sled along a frictionless snow path.

(a) (5 pts) He is applying a constant force $\overrightarrow{F}$ of magnitude 30 lbs along the rope which forms an angle of 10° with the horizontal path where the sled rests. What is the work done by this force when the sled is dragged over 20 ft? Your answer may still contain a trigonometric function but don't forget the overall unit.

(b) (5 pts) When Nanook reaches the dog yard, he runs to the cabin and pushes with his nose against the door with the same force, but at an angle of 60° to get it to open. What is the torque (direction and magnitude) about the hinge if Nanook pushes 8 in from the hinge? Simplify your answer.

Viewpoint is from the swallow nesting above the door: