MATH253X-UX1
Spring 2016

Midterm Exam 1

Name: _____

Instructions. (100 points) You have 60 minutes. No notes, book, or calculators allowed. *Show all your work* in order to receive full credit.

(16 pts) **1.** Consider a triangle in space with vertices $A(1, 0, -1)$, $B(2, 2, -2)$, and $C(-2, 1, 0)$.

(a) (3 pts) Find the equation of the sphere centered at A and going through B.

(b) (5 pts) Find the cosine of the angle at vertex A in the triangle.

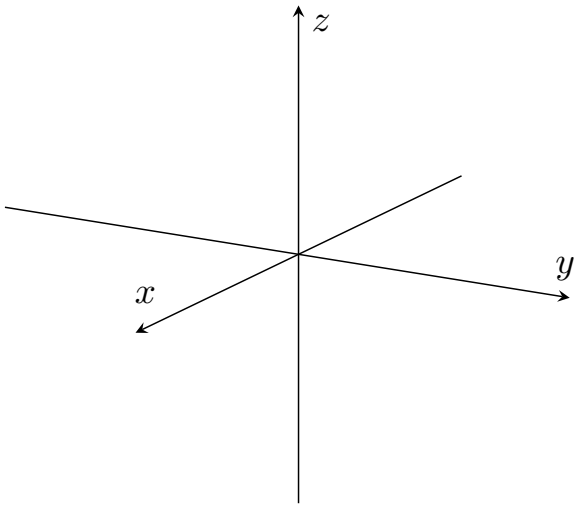
(c) (8 pts) Find the equation of the plane containing the triangle ABC.

(16 pts) **2.** Let $\mathbf{r}(t) = \langle 3\cos t, 5\sin t, -4\cos t \rangle$ describe the trajectory of a particle over time, where position is measured in meters and time in seconds.

(a) (6 pts) Find the distance traveled (i.e. the arc length) from $t = 0$ s to $t = 2\pi$ s.

(b) (4 pts) Show that the trajectory $\mathbf{r}(t) = \langle 3\cos t, 5\sin t, -4\cos t\rangle$ sits on both surfaces $4x + 3z = 0$ and $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$ at all times.

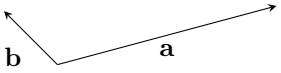
(c) (6 pts) Sketch the surface $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$. Include a scale.



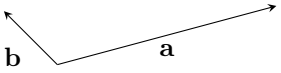
(8^{pts}) **3.** Consider vectors $\mathbf{a}$ and $\mathbf{b}$ as shown below.

(a) (4 pts) Sketch the following:

$\mathbf{a} - \mathbf{b}$



$\text{proj}_{\mathbf{b}}\,\mathbf{a}$



(b) (4 pts) State whether the following statements are true or false. Briefly justify.

- $\mathbf{a} \cdot \mathbf{b} \geq 0$
- $\mathbf{a} \times \mathbf{b}$ points out of the page towards you.

(14^{pts}) **4.** Let a space curve be described by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + 3t\mathbf{k}$.

(a) (8 pts) Find the symmetric equations of the tangent line to the curve at the point $P(-1, 1, -3)$.

(b) (6 pts) Find the tangential component of the acceleration for any t.

(10^{pts}) **5.** Consider the line given by:

$$x = 2 + t \quad , \quad y = 1 - t \quad , \quad z = 5 - 4t.$$

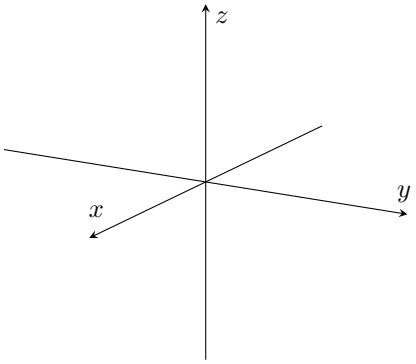
(a) (5 pts) Show that the line is parallel to but not in the plane $x - 3y + z = 1$.

(b) (5 pts) Find the distance from the line to the plane. *Hint:* You can use any point from the line.

(12$^{\text{pts}}$) **6.** Identify the surface from its equation then sketch the surface.

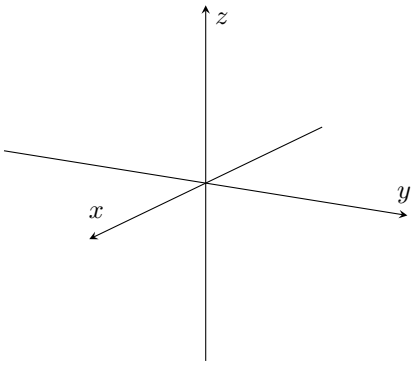
(a) (6 pts) $z = 1 + x^2 + \dfrac{y^2}{4}$

Type of surface: _____



(b) (6 pts) $x^2 + z^2 - 4y^2 = 4$

Type of surface: _____



(12$^{\text{pts}}$) **7.** A particle is moving in space from an initial position $\mathbf{r}(0) = \langle 0, 0, 1\rangle$ and initial velocity $\mathbf{v}(0) = \langle 2, 1, 2\rangle$ according to the following ***acceleration*** (measured in ft/s) at time t:

$$\mathbf{a}(t) = \left\langle 4e^{2t}, 6t, \frac{1}{(t+1)^2} \right\rangle \qquad , \qquad t \geq 0.$$

Find the position of the particle at $t = 1$ s.

(6pts) **8.** Let $f(t) = \dfrac{1}{t}$ and let $\mathbf{r}(t) = \langle t^2 - 1, \tan t \rangle$. Compute the derivative $\dfrac{d}{dt}[f(t)\mathbf{r}(t)]$ by using the rules of differentiation for a product. No credit will be given for substituting first.

(6pts) **9.** Physical applications. Choose *one* of the following problems to solve. You may do the other for extra credit only.

(a) Find the work done by gravity if a child on a sled with combined weight 60 lbs goes down 50 feet along a 30° incline.

(b) Find the magnitude of the torque when applying a force of 10 N directly upwards on a 20-cm wrench that makes a 45° angle with the horizontal.