MATH253X-UX1 Spring 2019

Final Exam

Name: ______________________

Instructions. (100 points) You have 120 minutes. Closed book, closed notes, no calculator. *Show all your work* in order to receive full credit.

(7^{pts}) **1.** Consider the following points in space: $A(-2,0,1)$, $B(1,1,-1)$, and $C(0,2,0)$.

(a) (3 pts) Find parametric equations for the line going through A and B.

(b) (4 pts) Find the area of the parallelogram with adjacent sides AB and AC.

(9^{pts}) **2.** Assume a particle has velocity $\mathbf{v}(t) = \langle 2t, t^2, 2 \rangle$ with speed measured in ft/s.

(a) (4 pts) Find the position vector $\mathbf{r}(t)$ at all times if $\mathbf{r}(2) = \langle 2, 3, 1 \rangle$.

(b) (5 pts) Find the distance traveled from $t = 1$ s to $t = 3$ s.

(10pts) **3.** Let $f(x,y) = x^2y^2 - xy^2 - x^2 - 2y^2 + x$.

(a) (7 pts) Verify that $(1/2, 0)$ and $(-1, 1)$ are (among the) critical points of $f(x,y)$. Then classify them using the Second Partials Test.

(b) (3 pts) Find the directional derivative of f when moving from $(0,2)$ towards $(-1,3)$.

(6pts) **4.** Switch the order of integration then compute

$$I = \int_0^4 \int_{y^{\frac{3}{2}}}^8 \sqrt{y} e^{x^2} \, dx \, dy$$

(9^{pts}) **5.** Consider a particle moving along C parametrized by $\mathbf{r}(t) = \langle t^2 - 1, 2t, t \rangle$, $1 \le t \le 2$ through the vector field $\mathbf{F}(x, y, z) = \langle 2xy - 1, x^2 - z, 2z - y \rangle$.

(a) (6 pts) The field is conservative. Find *all* potential functions.

(b) (3 pts) Apply the Fundamental Theorem of Line Integrals to compute the circulation (work).

(12^{pts}) **6.** Sketch the following:

(a) (6 pts) the surfaces $4x^2 + 9y^2 + z^2 = 9$ and $2x - 3y + 6z = 6$ and their intersection;

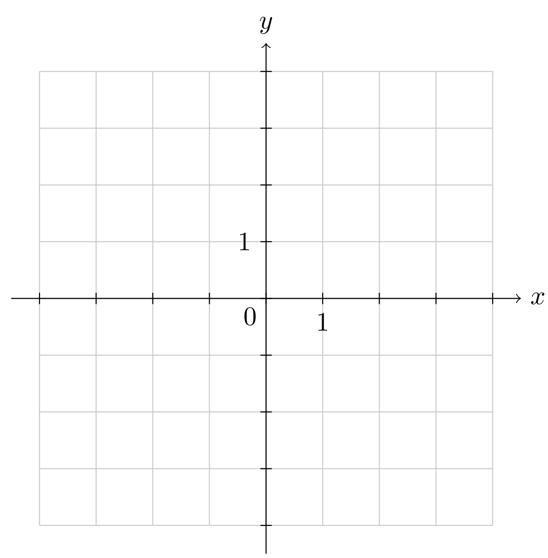
(b) (6 pts) the surface given in spherical coordinates by $\phi = \dfrac{\pi}{4}$, $-\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$, and $0 \le \rho \le 2 \sec \phi$.

(11 pts) **7.** Consider the hyperboloid of two sheets:

$$x^2 + 4y^2 - z^2 = -4$$

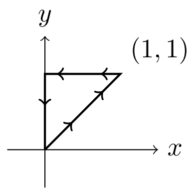
(a) (4 pts) Find an equation of the tangent plane to the hyperboloid at $(1, -1, 3)$.

(b) (3 pts) Sketch the level curves corresponding to $z = 2$ and $z = 2\sqrt{5}$.



(c) (4 pts) Fully SET UP an expression with triple integrals to represent $\bar{x}$ in the center of mass of the solid bounded by the hyperboloid and the plane $z = 2\sqrt{5}$ if the density of the solid is given by $\rho(x, y, z) = 2y^2 z$. DO NOT EVALUATE.

(6pts) **8.** Use Green's theorem to find the circulation of the vector field $\mathbf{F}(x,y) = \langle ye^x - \sin x, 2xy \rangle$ over the closed curve C described below:



(14pts) **9.** Let $f(x,y) = (x-1)^2 + 2y^2$.

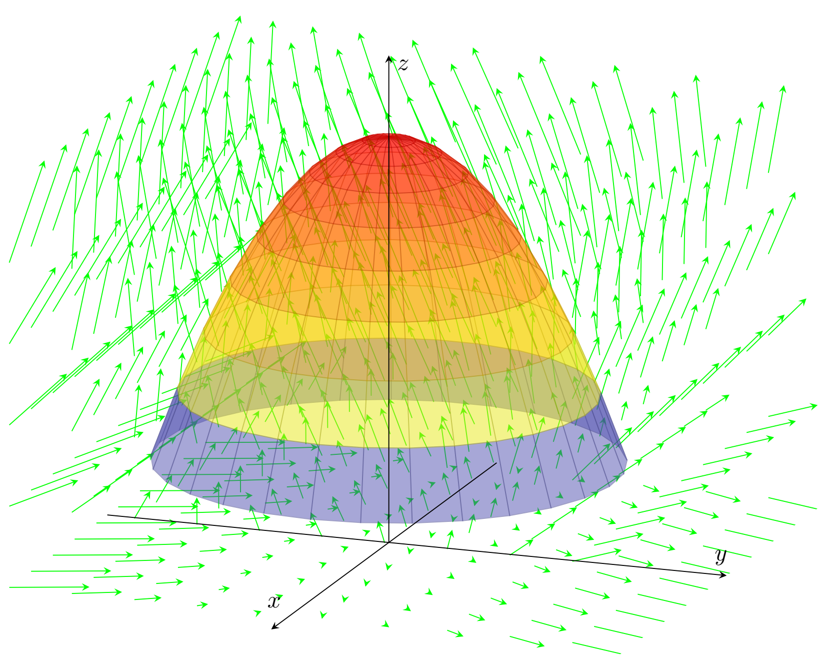
(a) (6 pts) Use the appropriate chain rule (not direct substitution) to find $\dfrac{\partial f}{\partial s}$ for $(s,t) = (2,-1)$ if $x = 2st$, $y = t^2 - s$.

(b) (8 pts) Use the gradient and Lagrange multipliers to find the absolute minimum and maximum of the function $f(x, y) = (x-1)^2 + 2y^2$ in the region $x^2 + y^2 \leq 4$.

(16^{pts}) **10.** Consider the surface S parametrized by

$$\mathbf{r}(u, v) = \langle u\cos v, u\sin v, 5 - u^2 \rangle \quad , \quad 0 \leq u \leq 2\,,\ 0 \leq v \leq 2\pi$$

and a vector field $\mathbf{F} = \langle y, y^2 - z, 3z \rangle$.



(a) (4 pts) Fully set up in (u, v) the flux of the curl across the surface oriented *upwards*. DO NOT evaluate.

$\iint_S \operatorname{curl} \mathbf{F} \cdot \mathbf{N} \, dS =$

(b) (6 pts) Stokes' theorem states that:

$$\iint_S \operatorname{curl} \mathbf{F} \cdot \mathbf{N} \, dS = \int_C \mathbf{F} \cdot d\mathbf{r}$$

for C the boundary curve of the surface S oriented here counterclockwise. Give a parametrization in t of C then use it to compute the line integral equivalent to the flux of the curl.

(c) (6 pts) Close the surface S by including the portion of the plane $z = 1$ that is on the bottom of S. Now use the divergence theorem (stated below) to compute the flux of the vector field across the new closed surface S' as a triple integral (use cylindrical coordinates). *Hint*: The original surface S satisfies $z = 5 - x^2 - y^2$.

$$\oiint_{S'} \mathbf{F} \cdot \mathbf{N}\, dS = \iiint_Q \operatorname{div} \mathbf{F}\, dV$$