MATH202X-F01-UX1
Spring 2015

Final Exam

Name:____________________________

Instructions. (100 points) You have 120 minutes. No calculators allowed. *Show all your work* in order to receive full credit.

(13^{pts}) **1.** A triangle has the following vertices

$$A = (2, 2, 1),\ B = (1, -1, 2),\ C = (0, 0, 0).$$

(a) (4 pts) Find the cosine of the angle at vertex B in the triangle. Is the angle acute, obtuse, or right?

(b) (3 pts) What is the area of the triangle ABC? (*Hint:* The area of a triangle is half of that of a parallelogram.)

(c) (3 pts) Give the equation of the plane containing ABC.

(d) (3 pts) Find the intersection of the plane containing ABC and the line through $(6, 0, 3)$ with direction $\mathbf{v} = \langle 2, -1, 1 \rangle$.

(4^{pts}) **2.** Use the chain rule to find $\dfrac{\partial f}{\partial s}$ where $f(x,y,z) = xy^2 + 2z$ and $(x,y,z) = (s-t, st, e^t)$. The final answer should be in terms of s and t only.

(4^{pts}) **3.** Consider the parameterized surface:

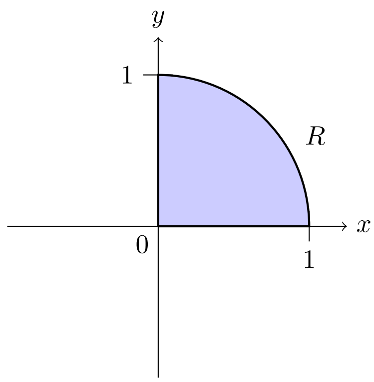
$$\mathbf{r}(u,v) = \langle 2u, v, u^2 \rangle \qquad , \qquad 0 \le u \le 1\ ,\ 0 \le v \le u.$$

Find the surface area of $\mathbf{r}(u,v)$.

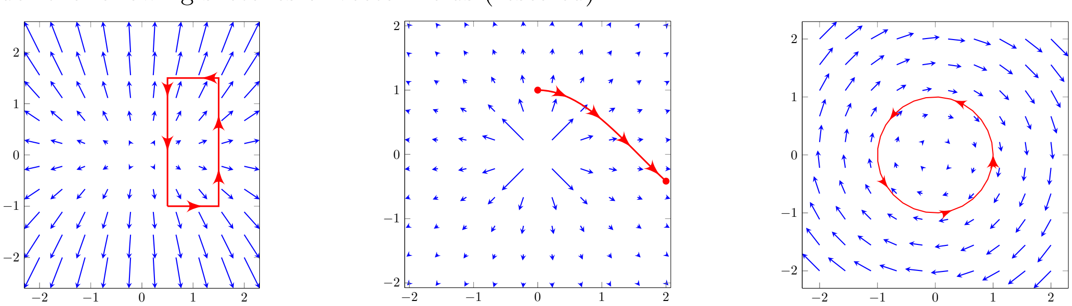
(8^{pts}) **4.** Find and classify all the critical points of

$$f(x,y) = \frac{1}{2}x^2 - xy + \frac{1}{3}y^3.$$

(8^{pts}) **5.** Find the center of mass of the planar lamina with density $\rho(x, y) = 4y$ described by the region R illustrated below. You may use that the mass of the lamina is $m = \frac{4}{3}$ without computing it.



(3^{pts}) **6.** Consider the following sketches of vector fields (rescaled).



(a) $\mathbf{F_1} = \langle x, 2y \rangle$ (b) $\mathbf{F_2} = \left\langle \dfrac{x}{x^2+y^2}, \dfrac{y}{x^2+y^2} \right\rangle$ (c) $\mathbf{F_3} = \langle y, -x \rangle$

Indicate whether work done by the vector field on a particle moving along the given curve will be positive, negative or zero. Briefly justify your choices.

(a)

(b)

(c)

(10pts) **7.** Consider the surfaces given by:

$$S_1 : \theta = \frac{\pi}{2} \quad \text{(cylindrical)} \qquad , \qquad S_2 : \rho = 3 \quad \text{(spherical)}$$

(a) (4 pts) Sketch and describe the surfaces and their intersection.

(b) (3 pts) Parametrize the surface S_2 giving bounds on the parameters.

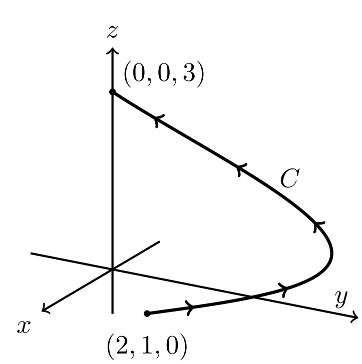
(c) (3 pts) Parametrize the intersection of S_1 and S_2 giving bounds.

(8pts) **8.** A force field is given by

$$\mathbf{F}(x, y, z) = \langle 2xy, x^2 + z\sin(yz), y\sin(yz) + 2z - 1\rangle.$$

(a) (4 pts) This field is conservative. Find *all* potential functions for it.

(b) (4 pts) How much work is done by $\mathbf{F}$ as a particle moves along the path C:



(5^{pts}) **9.** Reverse the order of integration of the following.

$$\int_0^{16}\int_{x/4}^{\sqrt{x}} f(x,y)\,dy\,dx$$

(6^{pts}) **10.** Suppose the temperature at a point (x, y) is given by

$$T = 100 - 0.5x^2 - y^2,$$

with x, y measured in meters, and T in degrees, and you are currently at the point $(3, 2)$. The positive x-axis points east, and the positive y-axis points north.

(a) (3 pts) If you walk northwest, will the temperature increase or decrease? At what rate?

(b) (3 pts) In which direction does the temperature increase most rapidly? At what rate?

(5^{pts}) **11.** Sketch the curve $\mathbf{r}(t) = \left\langle \cos t, \sin t, \frac{t}{2} \right\rangle$ for $0 \leq t \leq 6\pi$ and compute its length.

(12pts) **12.** Let the surface S be that part of the paraboloid $z = 4 - x^2 - y^2$ above the plane $z = 0$.

(a) (8 pts) Calculate the flux of $\mathbf{F} = \langle x, y, 0 \rangle$, through the surface S oriented using the upward pointing normals.

(b) (4 pts) Using the Divergence Theorem, calculate the flux of $\mathbf{F}$ through the closed surface given by S together with a disc in the plane $z = 0$. Recall that the Divergence Theorem states that, for appropriate S and Q,

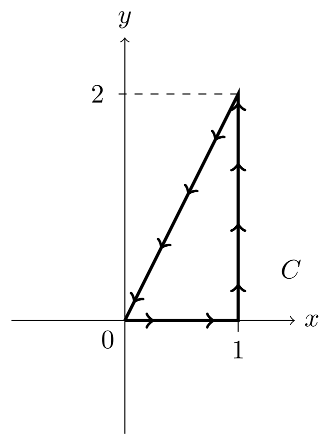
$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_Q \operatorname{div} \mathbf{F} \, dV.$$

(6pts) **13.** An object moves in space with velocity $\mathbf{v}(t) = \langle \sin t, 2\cos t, 1 \rangle$. At $t = 0$, it has position $\langle 0, 0, -1 \rangle$. Find the object's trajectory as a function of time.

(8pts) **14.** Throughout this problem, we consider the following line integral:

$$I = \oint_C (x-y)dx + xy^2 dy$$

where C is the closed curve shown below:



(a) (5 pts) Use Green's theorem to evaluate the line integral I.

(b) (3 pts) Use a parametrization to fully set up $\int_{C'} (x-y)\ dx + xy^2\ dy$ where C' is the diagonal segment of C. Do NOT evaluate.