Final Exam
Math 202 F01

Name:____________________
12/14/12

1. Suppose that the temperature T, in degrees Celsius, in a certain region of space is given by the function

$$T(x, y, z) = 5x^2 - 3xy + yz,$$

where the position coordinates x, y, z are in meters.

(a) (5 pts.) What is the directional derivative at $(2, 0, 3)$ in the direction towards $(3, -2, 3)$? *Indicate units for your answer.*

(b) (4 pts.) From the point $(2, 0, 3)$, in what direction should you begin moving to experience the greatest rate of *cooling*, and what would that rate be?

(c) (8 pts.) A straight wire is stretched from (2,0,3) to (1,1,1). Give an expression for the average temperature along the wire. Leave your answer in a form that a Calculus I student would understand; you do not need to completely evaluate any integrals.

2. (6 pts.) Find all critical points of $f(x, y) = x^3y + 12x^2 - 8y$, and, if possible, determine whether they are local maxima, local minima, or saddles.

3. Consider the vector field $\mathbf{F} = \langle y^2 + 6ye^{3x}, 2e^{3x} + y + 2xy \rangle$.

   (a) (4 pts.) This field is conservative. Find a potential function for $\mathbf{F}$.

   (b) (4 pts.) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the path parametrized by

$$\mathbf{r}(t) = \langle t\sin(\pi t), 2 + t\cos(\pi t) \rangle, \quad 0 \le t \le 4.$$

   (c) (2 pts.) If the field $\mathbf{F}$ represents a force, what is the physical interpretation of the integral you computed in part (b)?

4. A surface S is parameterized by $\mathbf{r}(u, v) = \langle u^2, u+v, u-v^2 \rangle$, $0 \leq u \leq 2$, $0 \leq v \leq 4$.

   (a) (4 pts.) Find an equation (of the form $ax + by + cz = d$) for the tangent plane to the surface at the point given by $u = 1, v = 2$.

   (b) (6 pts.) Give an integral that would compute the flux of the vector field $\mathbf{F} = \langle 0, x, -y \rangle$ through S, oriented so that the normal vector has a positive z-component. You may leave your answer as an iterated integral, provided all that remains to be done is evaluation of it.

5. (7 pts.) Find all points satisfying the constraint $x^2 + y^2 = 1$ at which the function $f(x, y) = x^2 + y$ has its maximum value.

6. (8 pts.) Let S be the closed surface whose bottom is the cone $z = \sqrt{x^2 + y^2}$ and whose top is the plane $z = 4$, oriented outward. Use Gauss's Divergence Theorem to compute the flux of the field

$$\mathbf{F} = \langle x + yz, x^2 + z^2, x^2 + z \rangle$$

through S.

7. (6 pts. – 3 pts. each)

(a) Draw a rough sketch of the level surface $w = 1$ of $w = f(x, y, z) = y^2 - z$.

(b) At the point $(1, 2, 3)$ on this level surface, find a *unit* normal vector.

8. (6 pts.) An object's velocity vector at time t is given by $\mathbf{v}(t) = \langle t^2, \sin t, 2 \rangle$, and its initial position at $t = 0$ is $\mathbf{r}(0) = \langle 1, 0, 2 \rangle$. Give a formula for its position at all times.

9. (7 pts.) Evaluate the following integral, by first expressing it in a different coordinate system:

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\, dz\, dy\, dx.$$

10. (8 pts.) Evaluate $\iint_R (x+1)\, dA$, where R is the region between the graphs of $y = x^2$ and $y = 2x$.

11. (15 pts. – 3 pts. each) Give short answers to the following:

(a) Give the equation of a plane through the point $(2, -1, 1)$ that is parallel to $3x - 2y + z = 1$.

(b) The area of a region R in the plane can be calculated by a line integral $\oint_C -\frac{y}{2}\,dx + \frac{x}{2}\,dy$. Where does this formula come from? What is C here and in what direction should it be followed?

(c) Is the angle between $\langle -1, 2, 3\rangle$ and $\langle 2, 3, -1\rangle$ acute ($< 90°$), right ($= 90°$), or obtuse ($> 90°$)? Show your work.

(d) If $\operatorname{curl} \mathbf{F} = 0$, then $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ for any closed loop C since...

(e) In polar coordinates, $dA = r\,dr\,d\theta$. Give a brief, informal indication of why the factor of 'r' appears in this formula.