Scalar triple product

Given $\vec{a}, \vec{b}, \vec{c}$ what does $\vec{a} \cdot (\vec{b} \times \vec{c})$ tell you?

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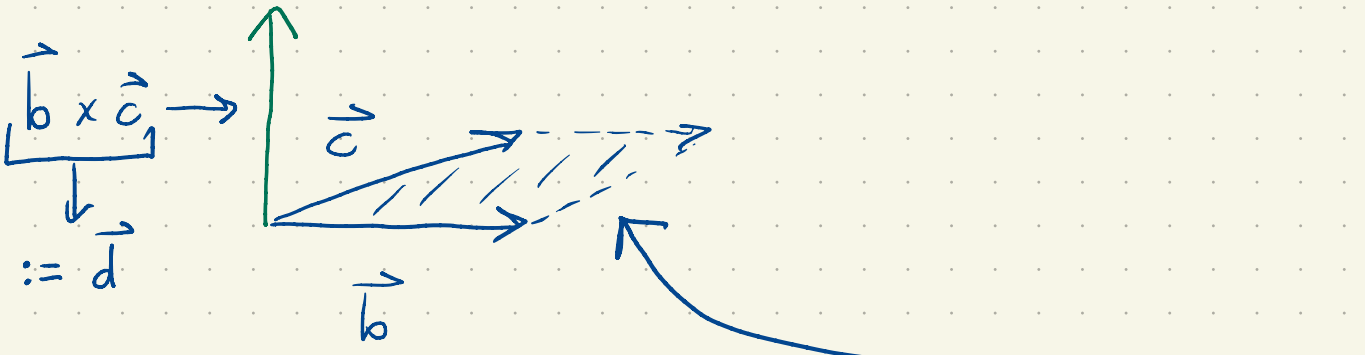
(1)



$\|\vec{c}\| = \|\vec{b}\| \, \|\vec{c}\| \sin\theta$ = Area of parallelogram

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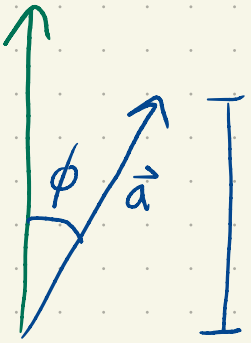
(2)

$\vec{b} \times \vec{c} = \vec{d}$



$\vec{a} \cdot \vec{d} = \|a\| \, \|d\| \cos\phi$

$h = \|\vec{a}\| \cos\phi$

$\Rightarrow \quad \vec{a} \cdot \vec{d} = \|d\| \, h$

(but h can be <u>negative</u> if $\vec{a}$ is left handed with respect to $\vec{b}, \vec{c}$)

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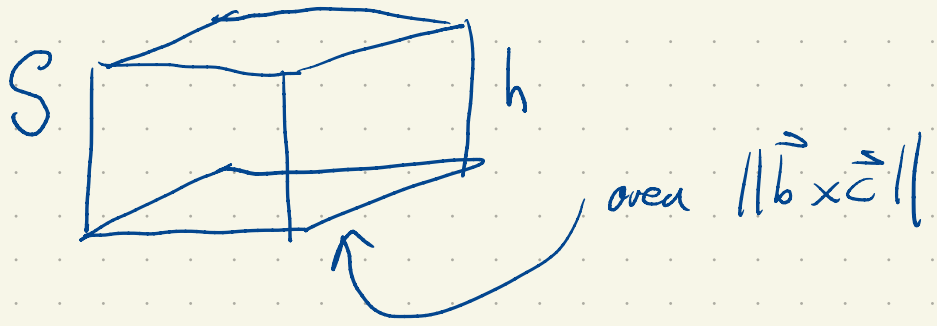
(3) Combine:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \|\vec{b} \times \vec{c}\| \, h$$



It is the volume of the solid S above, except it is negative if $(\vec{b}, \vec{c}, \vec{a})$ is left handed.

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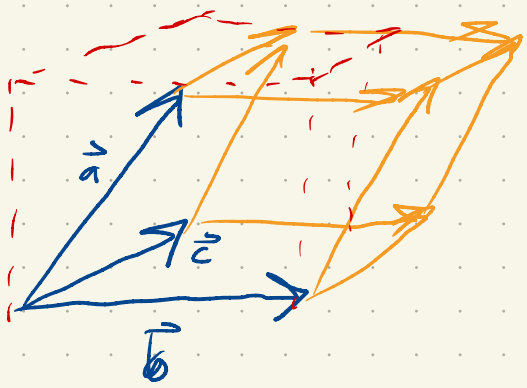
(4)



Volume of parallelepiped spanned by $(\vec{a}, \vec{b}, \vec{c})$ is same as volume of S

Conclusion:

$\vec{a} \cdot (\vec{b} \times \vec{c})$ is the volume of the parallelepiped spanned by $\vec{a}, \vec{b}, \vec{c}$ except it is positive if $(\vec{b}, \vec{c}, \vec{a})$ is right-handed and negative if $(\vec{b}, \vec{c}, \vec{a})$ is left handed.