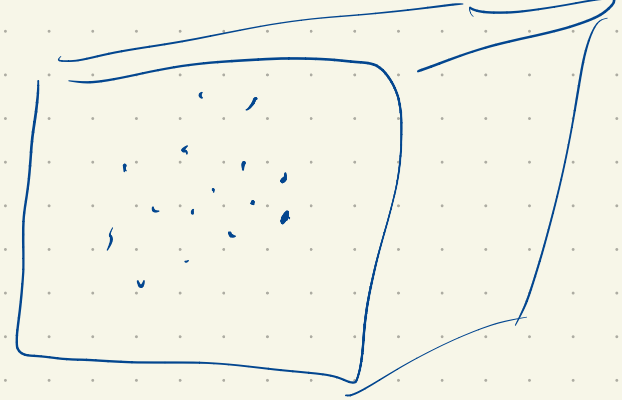
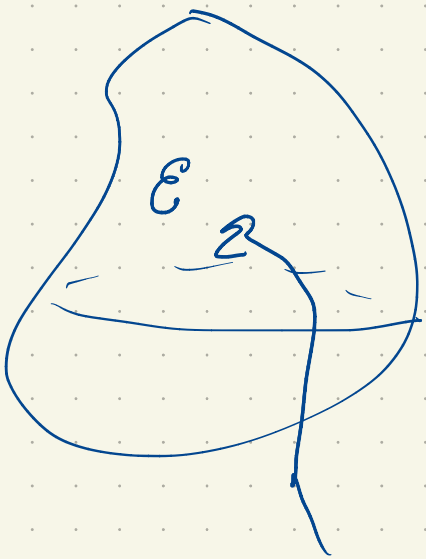
Charge



charge density

ρ $\qquad C/m^3$

$\hookrightarrow$ Coulombs measure charge

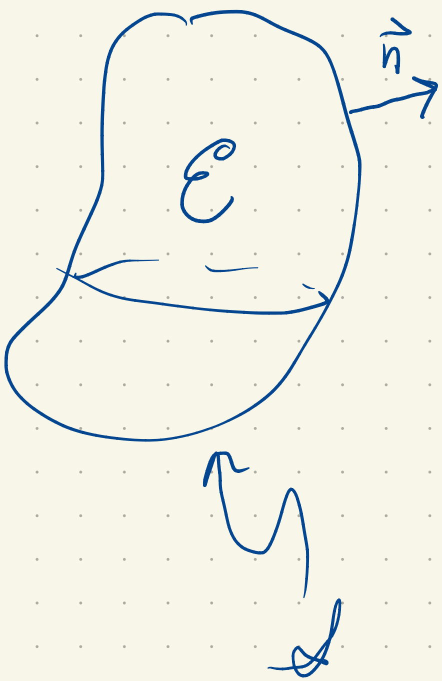


$$\iiint_{\mathcal{E}} \underset{C/m^3}{\rho}\, \underset{m^3}{dV} \qquad \underbrace{dx\,dy\,dz}$$

$\vec{v} \to$ velocity of moving charge

$$\vec{J} = \rho \vec{v} \qquad \frac{C}{m^3} \cdot \frac{m}{s} \qquad \frac{C}{m^2 s} = \frac{C}{s} \frac{1}{m^2}$$

$\vec{J} \to$ current density

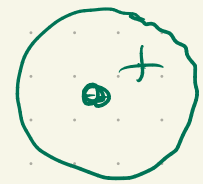


$$\iint_{\mathcal{S}} \vec{J} \cdot \vec{n} \, dA \qquad \frac{C}{s} \frac{1}{m^2} \cdot m^2 \qquad \frac{C}{s}$$

$\to$ rate at which charge is leaving $\mathcal{E}$.

$$\frac{d}{dt}\iiint_{E} \rho \, dV = -\iint_{S} \vec{J}\cdot\vec{n}\, dA$$

$$= -\iiint_{E} \vec{\nabla}\cdot\vec{J}\, dV$$



$$\iiint_{E} \frac{d\rho}{dt} + \vec{\nabla}\cdot\vec{J}\, dV = 0$$

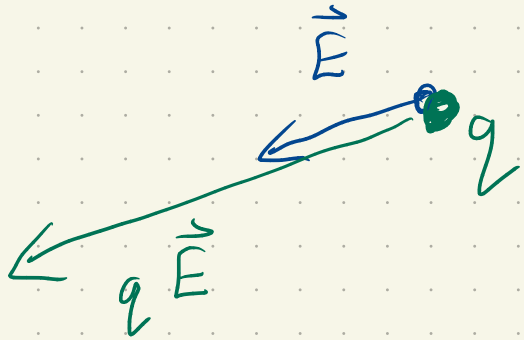
Partial differential equation

$$\boxed{\frac{d\rho}{dt} + \vec{\nabla}\cdot\vec{J} = 0}$$

continuity equation

Charge generates electric flux

$\vec{E}$ electric field $\qquad [\vec{E}] = \dfrac{N}{C}$ (force/charge)



$$\vec{F} = q\vec{E}$$

$$[\varepsilon_0] = \frac{C^2}{Nm^2}$$

Gauss'

$$\varepsilon_0 \overbrace{\iint_{\partial} \vec{E}\cdot\vec{n}\, dS}^{\text{electric flux}} = \iiint_{\mathcal{E}} \rho\, dV$$

$$\varepsilon_0 \iiint_{\mathcal{E}} \vec{\nabla}\cdot\vec{E}\, dV = \iiint_{\mathcal{E}} \rho\, dV$$

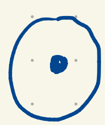
$$\iiint_{\mathcal{E}} \varepsilon_0 \vec{\nabla}\cdot\vec{E} - \rho \, dV = 0$$

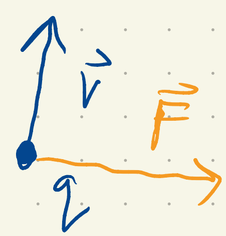
Gauss's law

$$\boxed{\varepsilon_0 \vec{\nabla}\cdot\vec{E} = \rho}$$

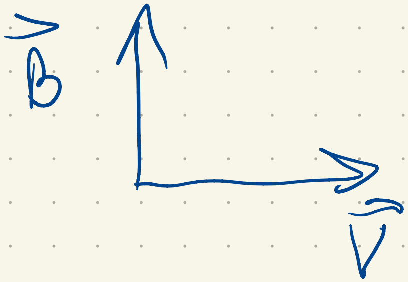
Magnetic field $\vec{B}$

1) Stationary charge is unaffected by $\vec{B}$.





$$\vec{F} = q\vec{v} \times \vec{B}$$

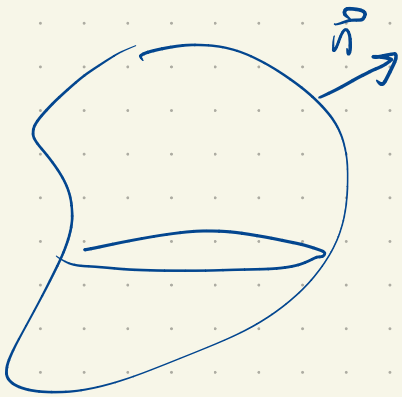


$$\vec{F} = q\vec{E}$$

$$[\vec{B}] = \frac{N}{C}\,\frac{s}{m}$$

"Gauss' Law for Magnetic Field"

The magnetic flux over every closed surface is zero.

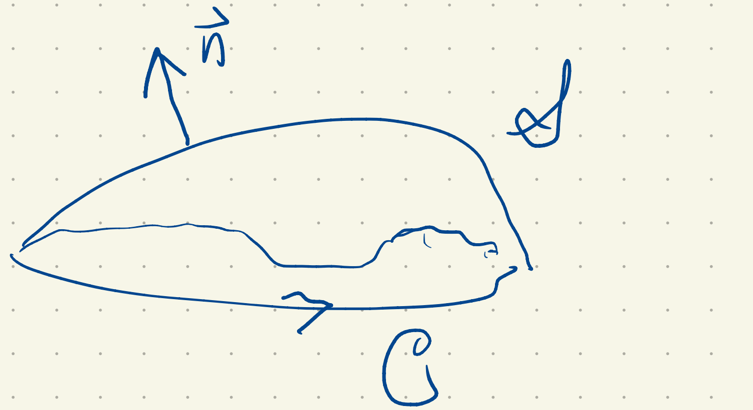


$$\iint_{S} \vec{B}\cdot\vec{n}\, dS = 0$$

$$\iiint_{E} \vec{\nabla}\cdot\vec{B}\, dV = 0$$

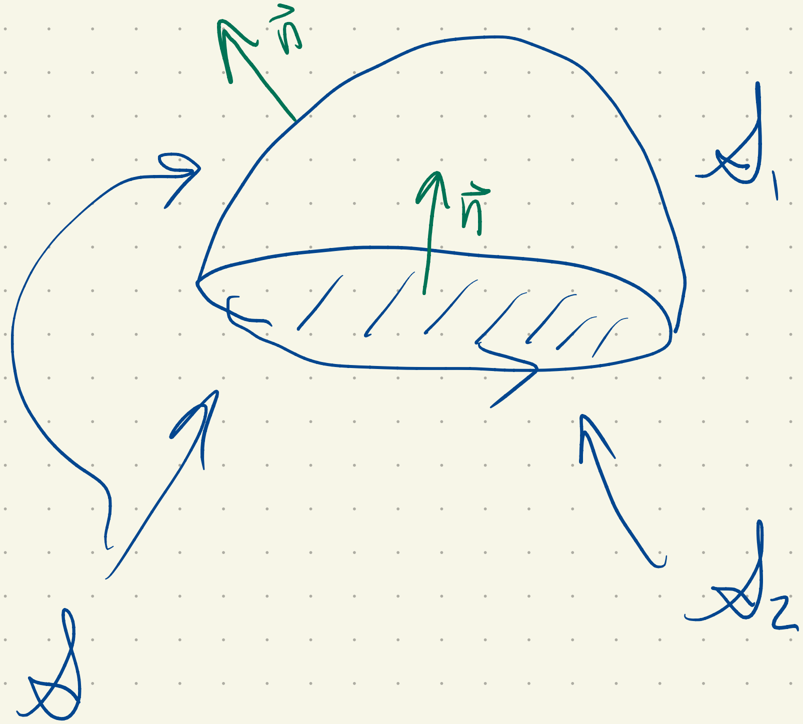
$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

Faraday's Law of Induction "changing magnetic fields induce current"



$$-\frac{d}{dt}\iint \vec{B}\cdot\vec{n}\,dS = \underbrace{\int_C \vec{E}\cdot d\vec{r}}_{\text{Voltage}} \qquad \frac{Nm}{C} = \frac{J}{C} = V$$

$$\int_C q\vec{E}\cdot d\vec{r} = \int_C \vec{F}\cdot d\vec{r}$$



$$\int_C \vec{E} \cdot d\vec{r} = -\frac{d}{dt} \iint_{S_1} \vec{B} \cdot \vec{n} \, dS$$

$$\int_C \vec{E} \cdot d\vec{r} = -\frac{d}{dt} \iint_{S_2} B \cdot \vec{n} \, dS$$

$$\frac{d}{dt} \iint_{S_1} B \cdot \vec{n} \, dS = \frac{d}{dt} \iint_{S_2} \vec{B} \cdot \vec{n} \, dS$$

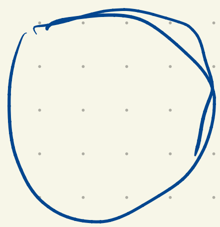
$$\frac{d}{dt} \left[\iint_{S_1} \vec{B} \cdot \vec{n} \, dS - \iint_{S_2} \vec{B} \cdot \vec{n} \, dS \right] = 0$$

$$\iint_{\mathcal{S}} \vec{B} \cdot \vec{n} \, dS = 0$$

$$-\frac{d}{dt} \iint \vec{B} \cdot \vec{n} \, dS = \int_C \vec{E} \cdot d\vec{r}$$

$$-\frac{d}{dt} \iint_{\mathcal{S}} \vec{B} \cdot \vec{n} \, dS = \iint_{\mathcal{S}} \vec{\nabla} \times \vec{E} \cdot \vec{n} \, dS$$

$$\iint_S \left[-\frac{d}{dt}\vec{B} + \vec{\nabla}\times\vec{E} \right] \cdot \vec{n}\, dS = 0$$

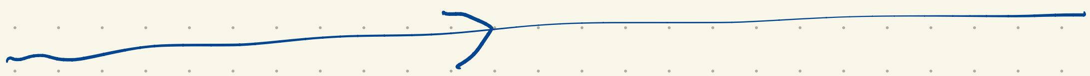


$$-\frac{d}{dt}\vec{B} + \vec{\nabla}\times\vec{E} = 0$$

$$\boxed{\frac{d}{dt}\vec{B} = \vec{\nabla}\times\vec{E}}$$

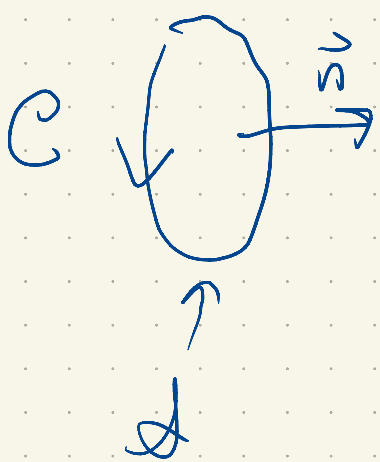
Amere's Law

Current generates a magnetic field



current (in amps) flowing through S

$$\iint_S \vec{J}\cdot\vec{n}\, dS$$



$$[\mu_0] = \frac{N}{C^2} s^2$$

$$\int_C \vec{B}\cdot d\vec{r} = \mu_0 \iint_S \vec{J}\cdot\vec{n}\, dS$$

$$\int_C \vec{B} \cdot d\vec{r} = \iint_S \vec{\nabla} \times \vec{B} \cdot \vec{n} \, dS$$

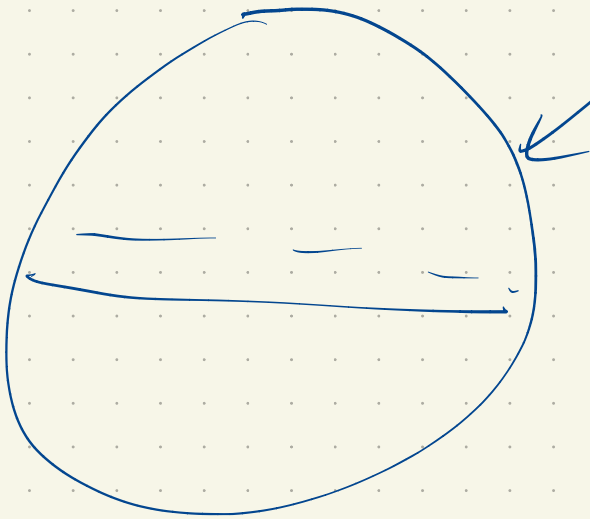
$$\iint_S (\vec{\nabla} \times \vec{B}) \cdot \vec{n} \, dS = \iint_S \mu_0 \vec{J} \cdot \vec{n} \, dS$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = 0 \rightarrow \vec{\nabla} \cdot (\mu_0 \vec{J}) = 0$$

$$\vec{\nabla} \cdot \vec{J} = 0$$

$$\frac{d}{dt}\rho + \vec{\nabla}\cdot\vec{J} = 0 \Rightarrow \frac{d}{dt}\rho = 0$$



S

empty boundary

C

$$\int_C \vec{B}\cdot d\vec{r} = 0$$

C

$$\iint_{\cancel{\partial\Omega}} J\cdot\vec{n} = 0 \quad , \text{not always}$$

S

$$\iint_{S\ \cancel{\partial\Omega}} \vec{J}\cdot\vec{n}\, dS = -\frac{d}{dt}\iiint_{\cancel{\Omega}\, E} \rho \, dV$$

$$= \iiint_{\mathcal{E}} -\frac{d}{dt} \varepsilon_0 \vec{\nabla} \cdot \vec{E} \, dV$$

$$= -\frac{d}{dt} \iint_{\mathcal{S}} \varepsilon_0 \vec{E} \cdot \vec{n} \, dS$$

$$\iint_{\mathcal{S}} (\vec{J} + \varepsilon_0 \vec{E}) \cdot \vec{n} \, dS = 0$$

$$\int_{\mathcal{C}} \vec{B} \cdot d\vec{r} = \iint_{\mathcal{S}} (\vec{J} + \varepsilon_0 \vec{E}) \cdot \vec{n} \, dS$$

$$\iint_S (\vec{\nabla} \times \vec{B}) \cdot \vec{n}\, dS = \mu_0 \iint_S (\vec{J} + \varepsilon_0 \vec{E}) \cdot \vec{n}\, dS$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \vec{E} \right)$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + (\mu_0 \varepsilon_0) \vec{E}$$

$$\frac{s^2}{m^2}$$

$[c]$ m/s

speed of light

$$= \mu_0 \vec{J} + \frac{1}{c^2} \vec{E}$$

$$\rho_t + \vec{\nabla} \cdot \vec{J} = 0$$

$$\varepsilon_0 \vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = \frac{d}{dt} \vec{B}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{d}{dt} \vec{E}$$