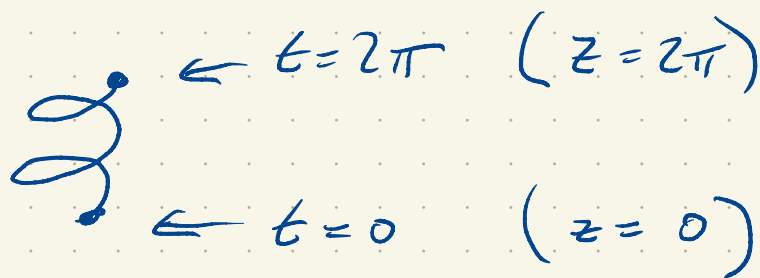




$$\vec{r}(t) = \langle \cos(2t), \sin(2t), t \rangle \quad 0 \leq t \leq 2\pi$$



$$\vec{r}(s) = \langle \cos(s), \sin(s), s/2 \rangle \quad 0 \leq s \leq 4\pi$$

$$\vec{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$$

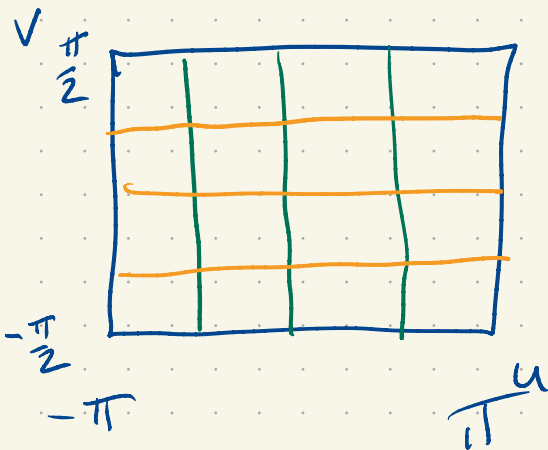
"parameterized surface"

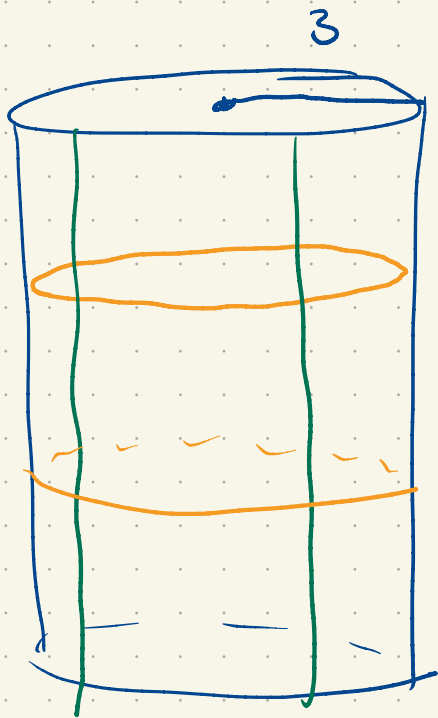
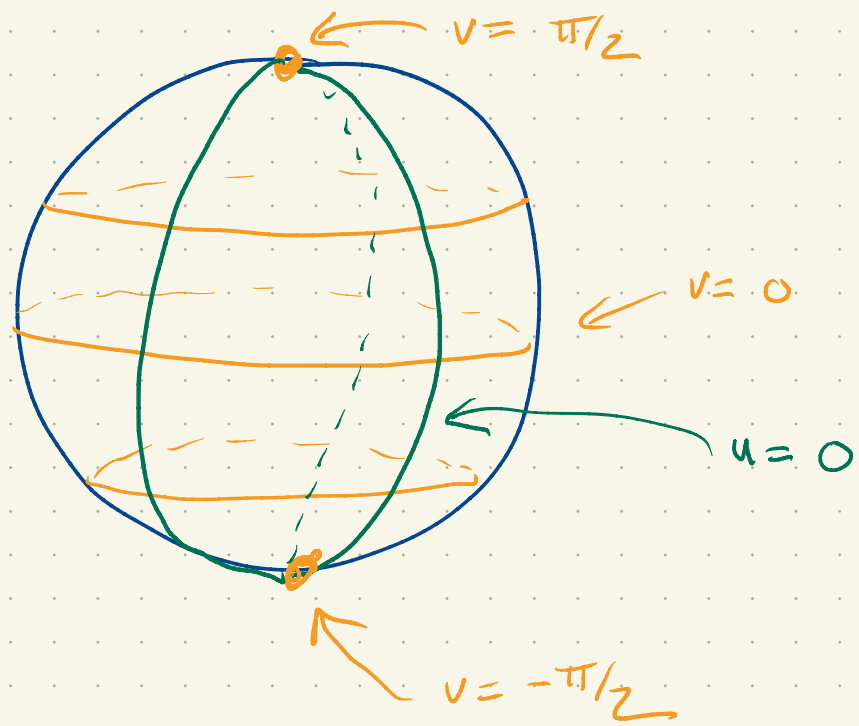
$$\vec{r}(u, v) = \langle \underbrace{\cos u \cos v}_{x(u, v)}, \sin u \cos v, \sin v \rangle$$

$\langle \cos v, 0, \sin v \rangle$

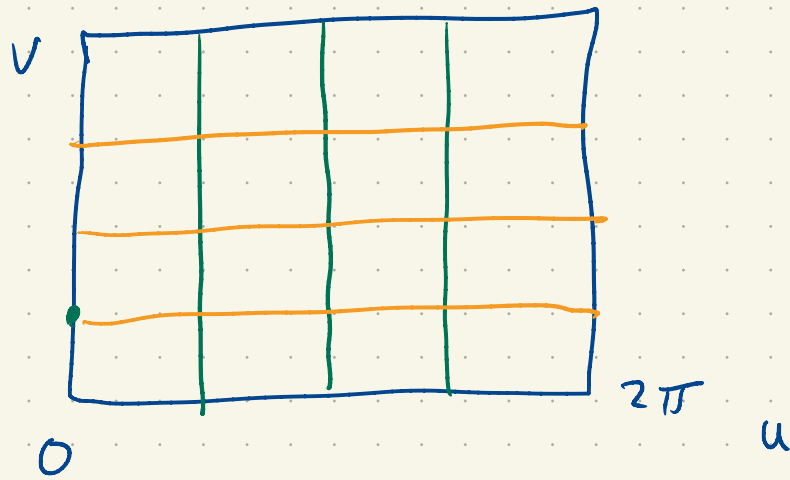
$$x^2 + y^2 = \cos^2 u \cos^2 v + \sin^2 u \cos^2 v$$
$$= \cos^2 v$$

$$x^2 + y^2 + z^2 = \cos^2 v + \sin^2 v = 1$$

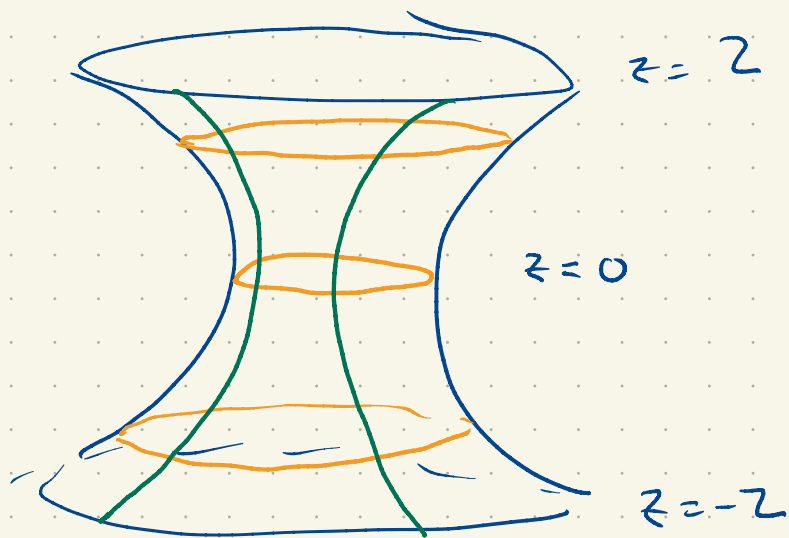




$$x^2 + y^2 = 9$$

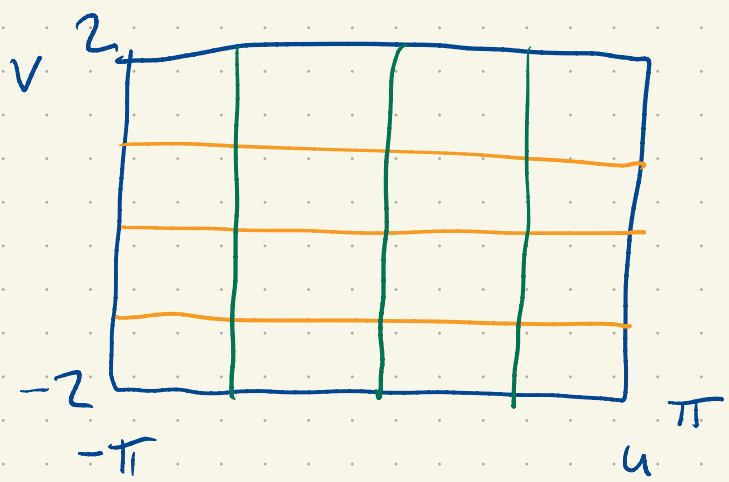


$$\vec{r}(u, v) = \langle 3\cos(u), 3\sin(u), v \rangle$$



$$x^2 + y^2 = z^2 + 1$$

$$r = \sqrt{z^2 + 1}$$



$$\vec{r}(u, v) = \langle r \cos u, r \sin u, v \rangle$$

$$\int_{-2}^2 \int_{-\pi}^{\pi} \text{[Diagram of a circle in the uv-plane]} du dv$$

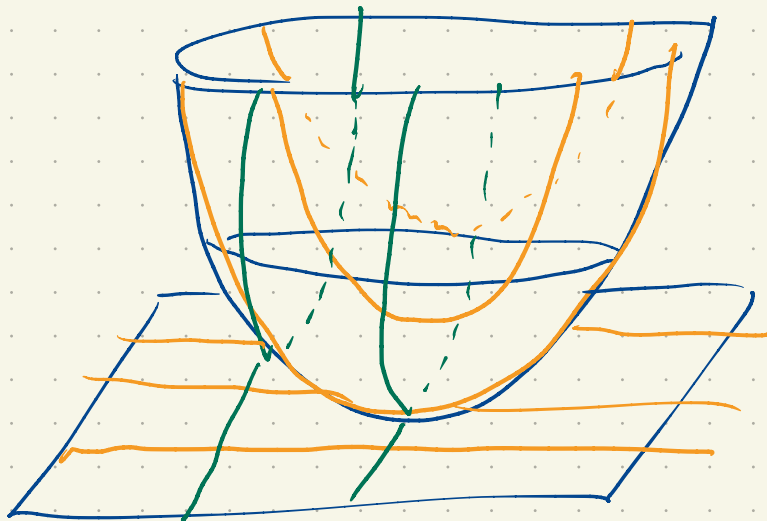
$$= \langle \sqrt{v^2 + 1} \cos u, \sqrt{v^2 + 1} \sin u, v \rangle$$

$$x^2 + y^2 = (v^2 + 1) \cos^2 u + (v^2 + 1) \sin^2 u = v^2 + 1$$

$$z^2 + 1 = v^2 + 1 \quad \leftarrow \quad = \checkmark$$

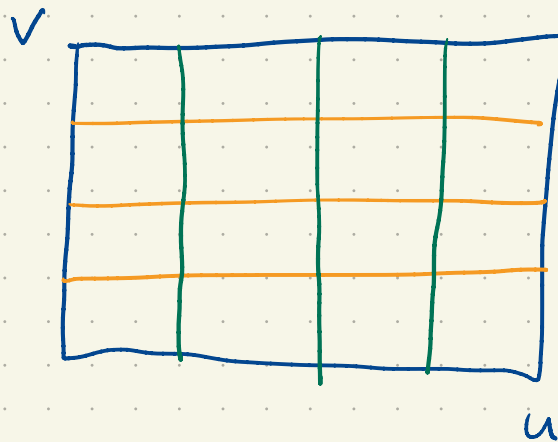
$$z = f(x, y)$$

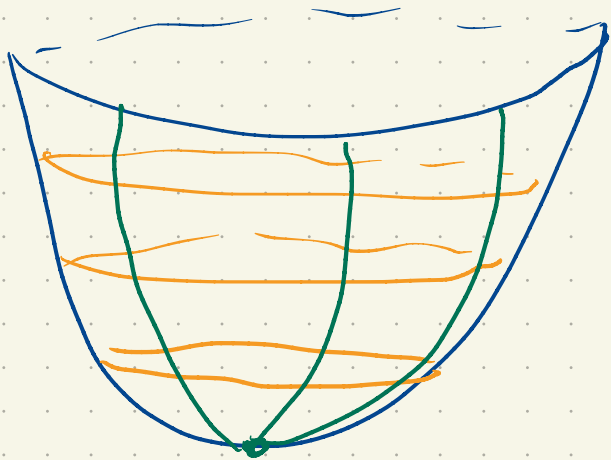
$$z = x^2 + y^2$$



$$\vec{r}(u, v) = \langle u, v, f(u, v) \rangle$$

$$\vec{r}(u, v) = \langle u, v, u^2 + v^2 \rangle$$

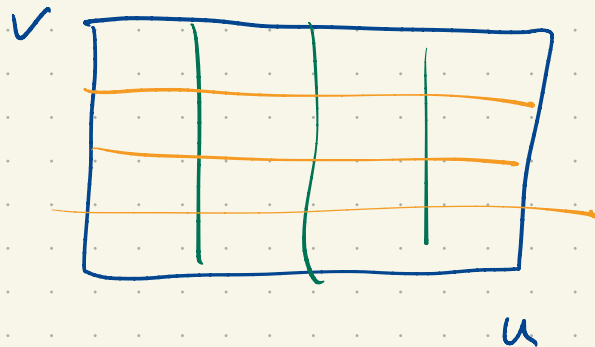




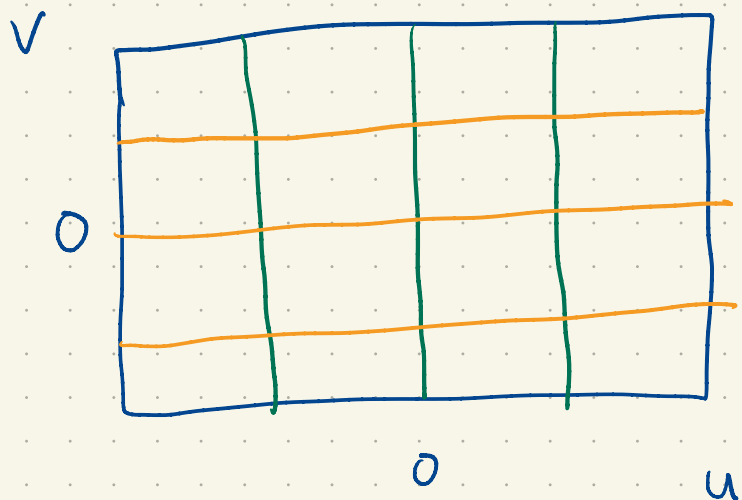
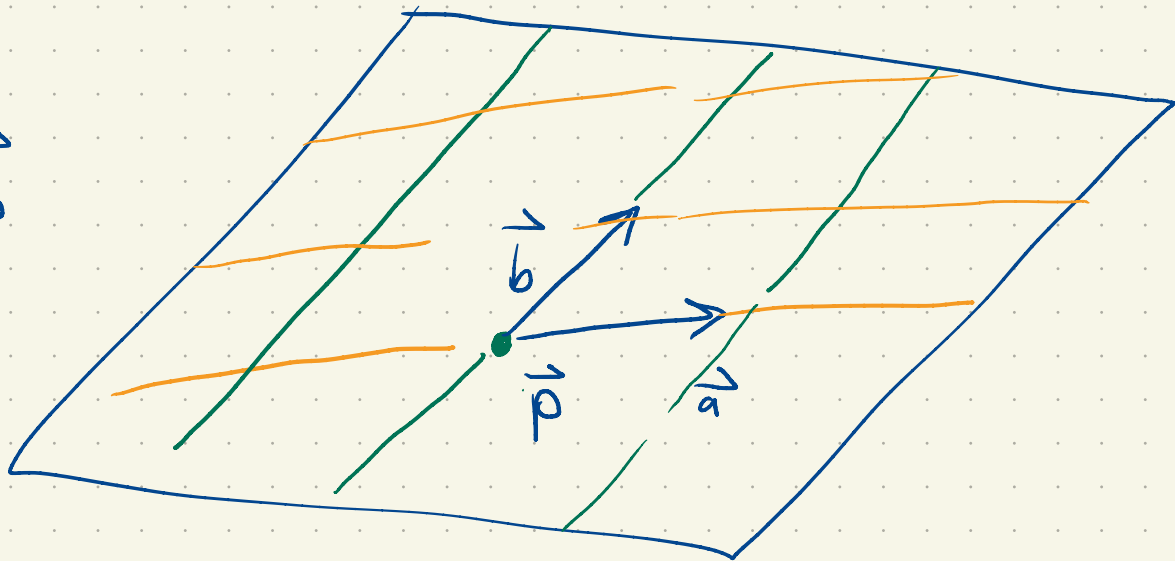
$$z = x^2 + y^2$$

$$\vec{r}(u, v) = \langle \sqrt{v} \cos u, \sqrt{v} \sin u, v \rangle$$

$$x^2 + y^2 = v \cos^2 u + v \sin^2 u = v = z \checkmark$$

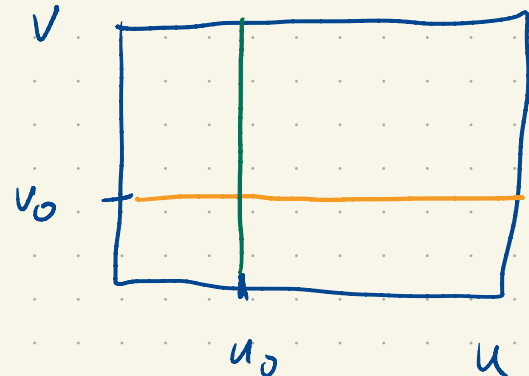
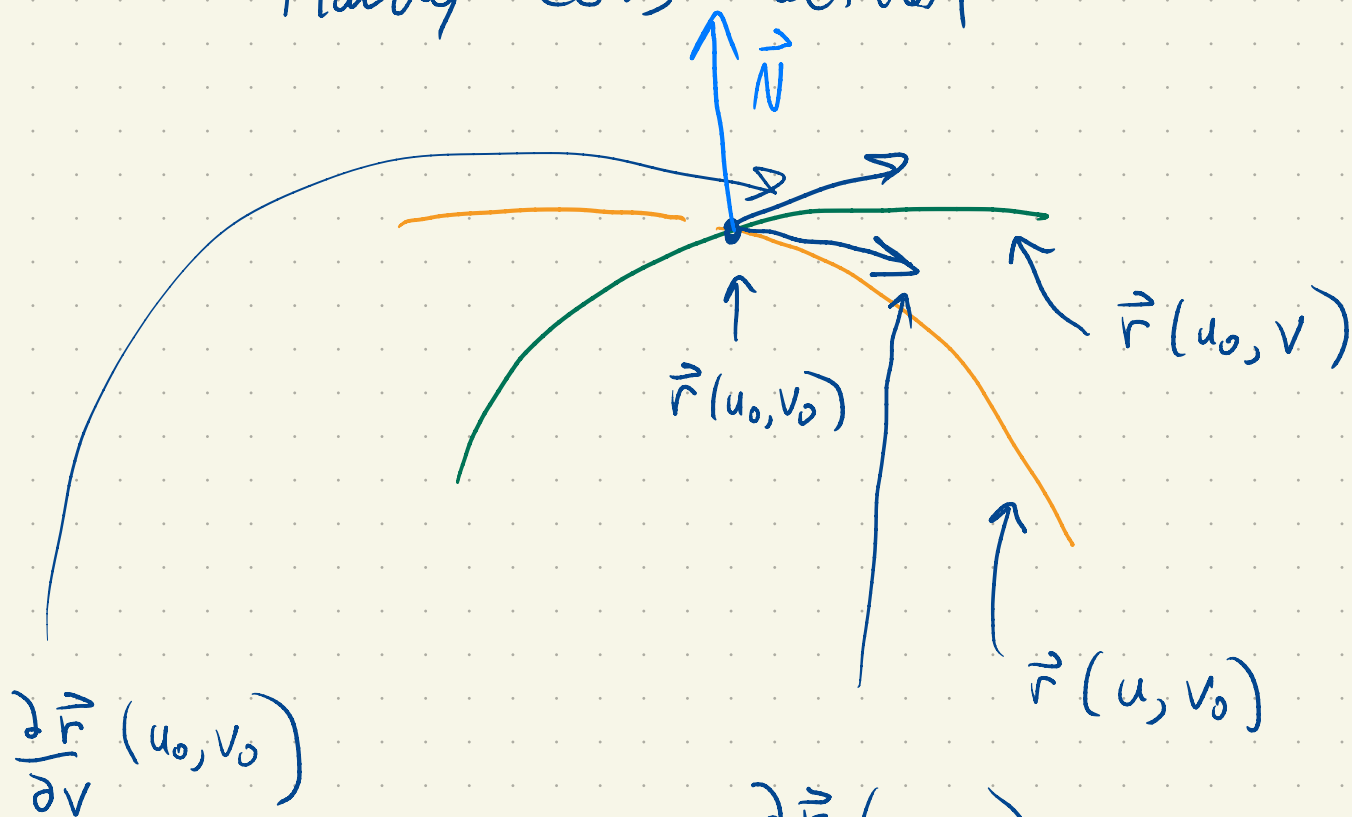


$$\vec{r}(u,v) = \vec{p} + u\vec{a} + v\vec{b}$$



$$\vec{r}(t) = \vec{p} + t\vec{v}$$

Handy Construction



$$\frac{\partial \vec{r}}{\partial v}(u_0, v_0)$$

$$\frac{\partial \vec{r}}{\partial u}(u_0, v_0)$$

$$\frac{\partial x}{\partial u}(u_0, v_0)$$

$$\left\langle \frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right\rangle$$

$$\vec{N} = \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}$$

$$\vec{r}(u, v) = \langle \cos u, \sin u, v \rangle$$

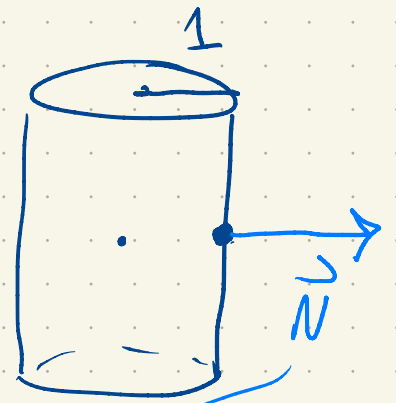
$$\frac{\partial \vec{r}}{\partial u} = \langle -\sin u, \cos u, 0 \rangle$$

$$\frac{\partial \vec{r}}{\partial v} = \langle 0, 0, 1 \rangle$$

$$\vec{N} = \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\sin u & \cos u & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \langle \cos u, \sin u, 0 \rangle$$

$$\vec{N}(0, 0) = \langle 1, 0, 0 \rangle$$



$$u=0, v=0$$

$$\langle 1, 0, 0 \rangle$$

parallel to

$$\langle 1, 0, 0 \rangle$$