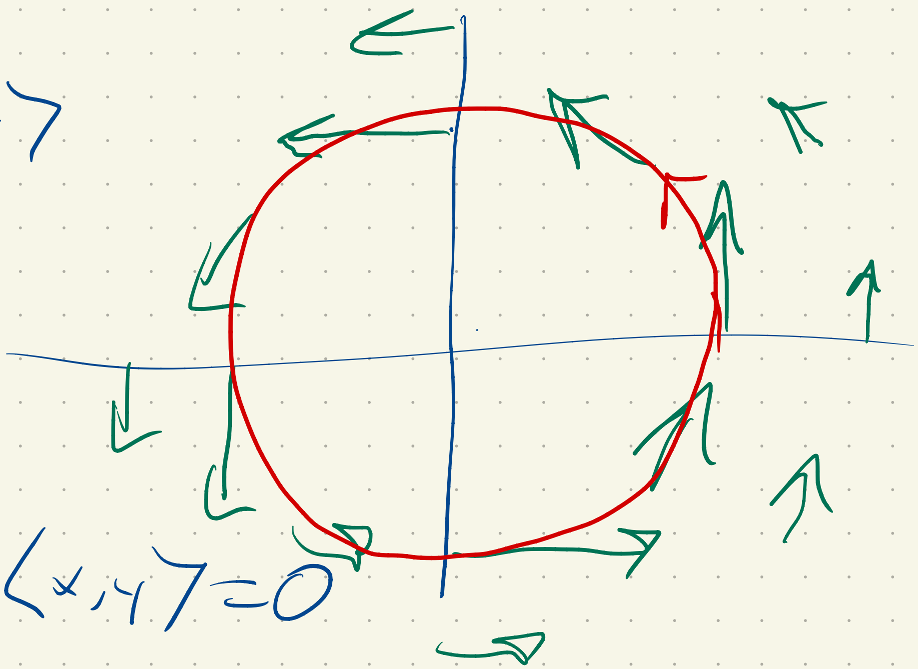
It is almost true that if $P_y = Q_x$ then $\vec{F}$ is conservative.

$\vec{F} = \langle P, Q \rangle$ $\quad P = \frac{-y}{x^2+y^2}$ $\quad Q = \frac{x}{x^2+y^2}$

$\int_C f \, ds$

$\int_C \vec{F} \cdot d\vec{r}$

$\langle x, y \rangle$



$\langle -y, x \rangle \cdot \langle x, y \rangle = 0$

$\oint_C \vec{F} \cdot d\vec{r} > 0$

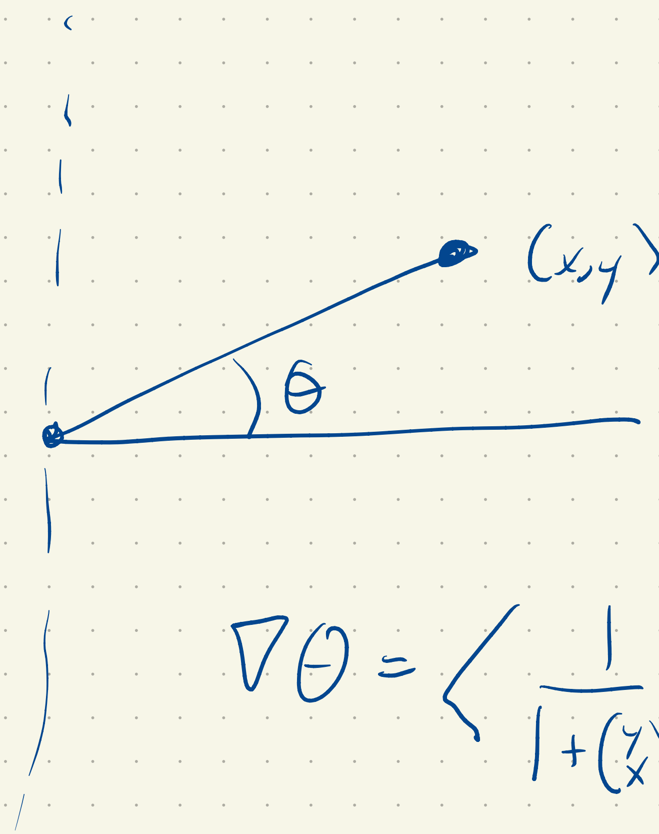
$\oint_C \vec{F} \cdot d\vec{r} = 0$ if $\vec{F}$ is cons.

$$P = \frac{-y}{x^2+y^2} \qquad Q = \frac{x}{x^2+y^2}$$

$$\frac{\partial P}{\partial y} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$\frac{\partial Q}{\partial x} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$P_y = Q_x$$



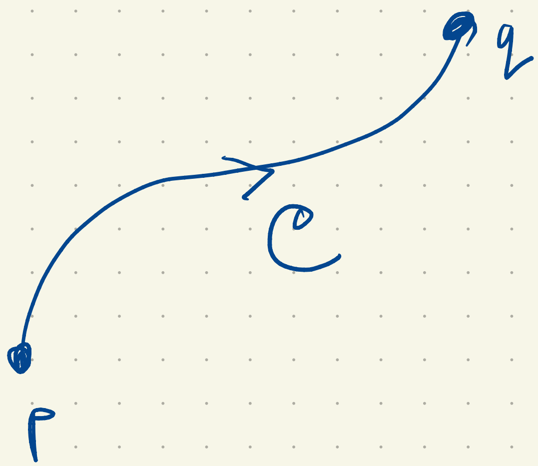
$$\theta = \arctan\left(y/x\right)$$

$$\nabla\theta = \left\langle \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{-y}{x^2},\ \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} \right\rangle$$

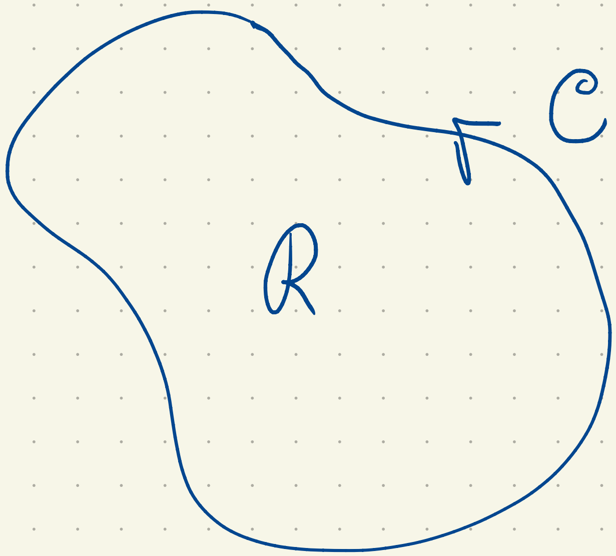
$$= \left\langle \frac{-y}{x^2+y^2},\ \frac{x}{x^2+y^2} \right\rangle$$

Green's Theorem

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$



$$\int_C \nabla f \cdot d\vec{r} = f(q) - f(p)$$



$$\iint_R \left(-\frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x}\right) dA = \oint_C \vec{F} \cdot d\vec{r}$$

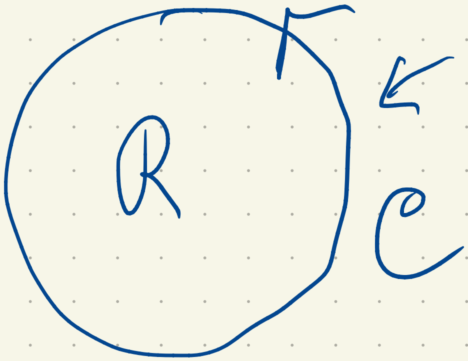
$$= \int_C P\,dx + Q\,dy$$

$$\vec{F} = \langle P, Q \rangle$$

$$P_y = Q_x$$

$$-P_y + Q_x = 0$$

$P = -x^2 y \qquad Q = x^3$



$x^2 + y^2 = 2$

$\vec{r}(t) = \langle \sqrt{2}\cos t, \sqrt{2}\sin t \rangle$

$$-\frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x} = +x^2 + 3x^2 = 4x^2$$

$$\iint_R -\frac{\partial P}{\partial y} + \frac{\partial Q}{\partial x}\, dA = \iint_R 4x^2\, dA$$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}} 4(r\cos\theta)^2\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^{\sqrt{2}} 4r^3 \cos^2\theta \, dr \, d\theta$$

$$= \int_0^{2\pi} r^4 \Big|_0^{\sqrt{2}} \cos^2\theta \, d\theta$$

$$= \int_0^{2\pi} 4 \cos^2\theta \, d\theta$$

$$= 4 \int_0^{2\pi} \frac{\cos(2\theta) + 1}{2} \, d\theta$$

$$= 2 \left(0 + \cdot 2\pi \right)$$

$$= 4\pi$$

$P = -x^2y \qquad Q = x^3$

$\vec{r} = \langle \sqrt{2}\cos t, \sqrt{2}\sin(t) \rangle$

$0 \le t \le 2\pi$

$$\oint_C P\,dx + Q\,dy$$

$$= \int_0^{2\pi} -2\cos^2 t\,\sqrt{2}\sin(t)\,\frac{dx}{dt} + 2\sqrt{2}\cos^3 t\,\frac{dy}{dt}\,dt$$

$$= \int_0^{2\pi} -2\sqrt{2}\cos^2 t\sin(t)\left(-\sqrt{2}\sin t\right) + 2\sqrt{2}\cos^3 t\,\sqrt{2}\cos(t)\,dt$$

$$= 4\int_0^{2\pi} \cos^2 t\sin^2 t + \cos^4 t\;dt$$

$$= 4\int_0^{2\pi} \cos^2 t\left(\sin^2 t + \cos^2 t\right)dt$$

$$= 4 \int_0^{2\pi} \cos^2 t \, dt$$

$$= 4 \int_0^{2\pi} \frac{\cos(2t) + 1}{2} \, dt$$

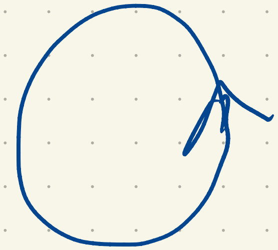
$$= 4 \int_0^{2\pi} \frac{1}{2} \, dt$$

$$= 4 \cdot 2\pi \cdot \frac{1}{2} = 4\pi$$

Fine print:

a-1) C is a loop

a) Orientation matters



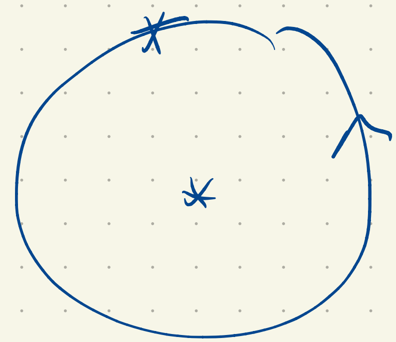
C is oriented counter clockwise.

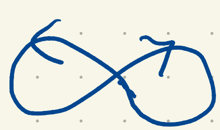
b) C is the entire boundary.

(R has no holes "simply connected")

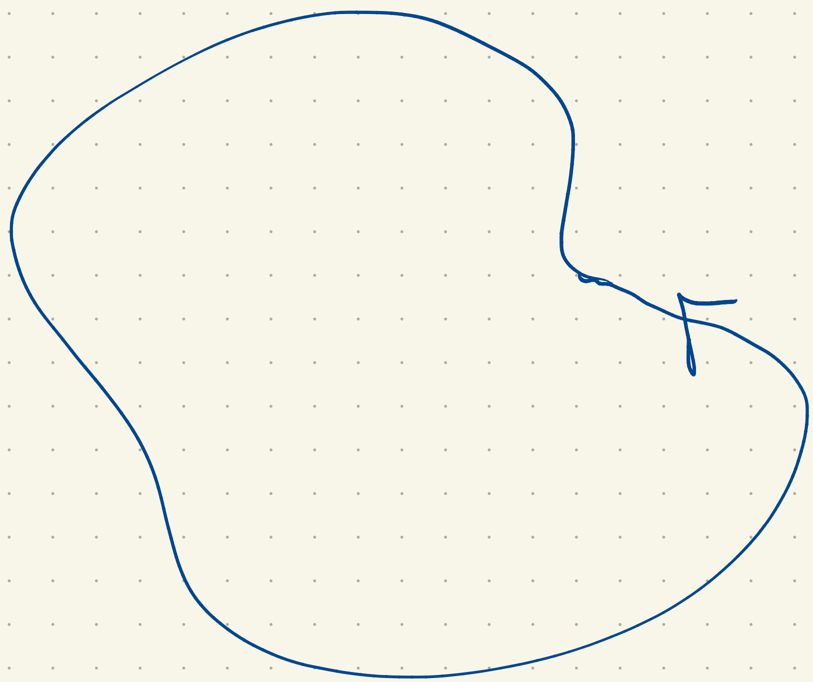
C

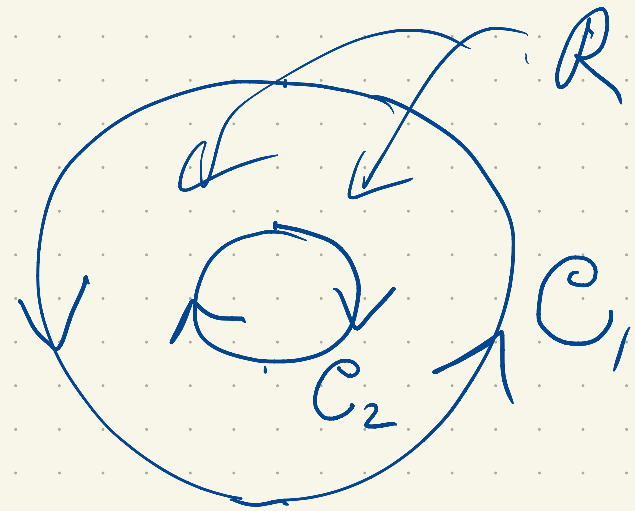
c) C has no self intersections

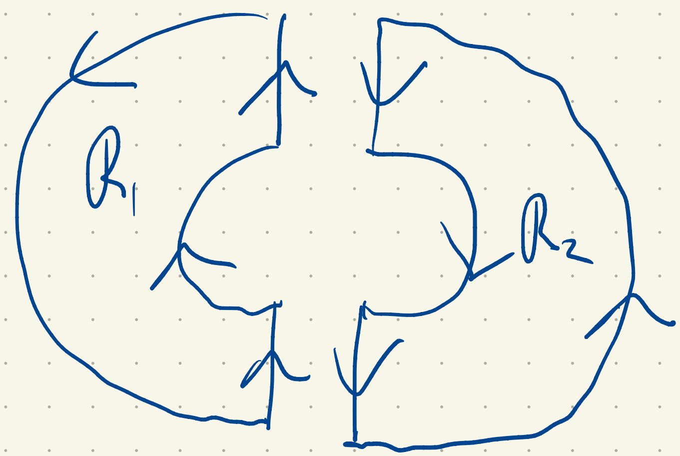




no no.



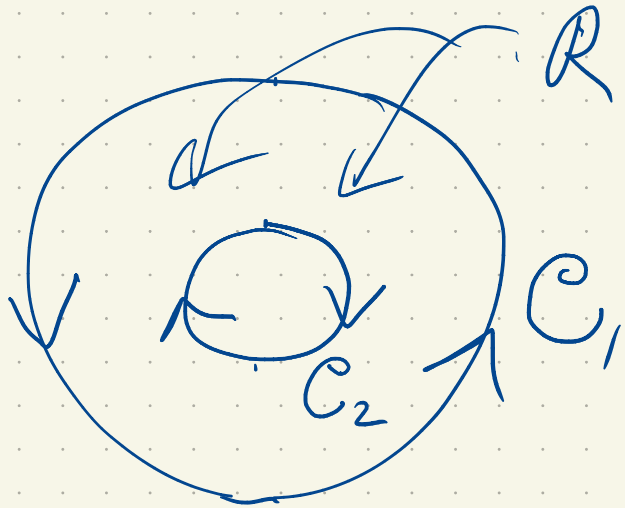


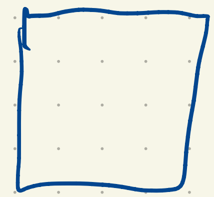


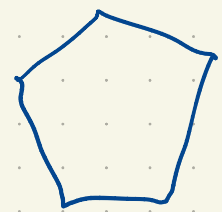
$$\iint_R -P_y + Q_x \, dA$$

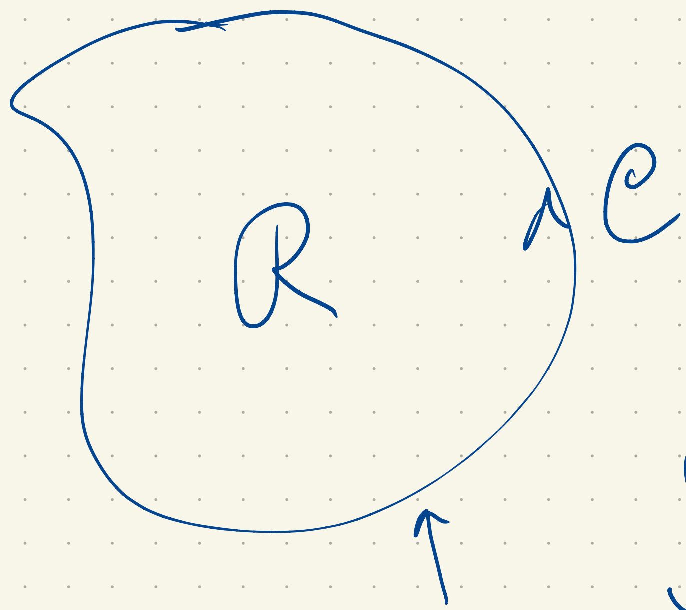
$$= \iint_{R_1} -P_y + Q_x \, dA + \iint_{R_2} -P_y + Q_x \, dA$$

$$\iint_R -P_y + Q_x \, dA = \int_{C_1} P\,dx + Q\,dy + \int_{C_2} P\,dx + Q\,dy$$









$P = -y$

$Q = 0$

$P\,dx + Q\,dy$

$\downarrow$

$$\int_C -y\,dx = \iint_R -P_y + Q_x \, dA$$

$$= \iint_R 1 \, dA = \iint_R dA$$

$P = 0$

$Q = x$

$$\int_C x\,dy = \iint_R -P_y + Q_x \, dA = \iint_R 0 + 1 \, dA = \iint_R dA$$

$$\frac{1}{2}\int_{C} -y\,dx + x\,dy = \iint_{R} dA$$