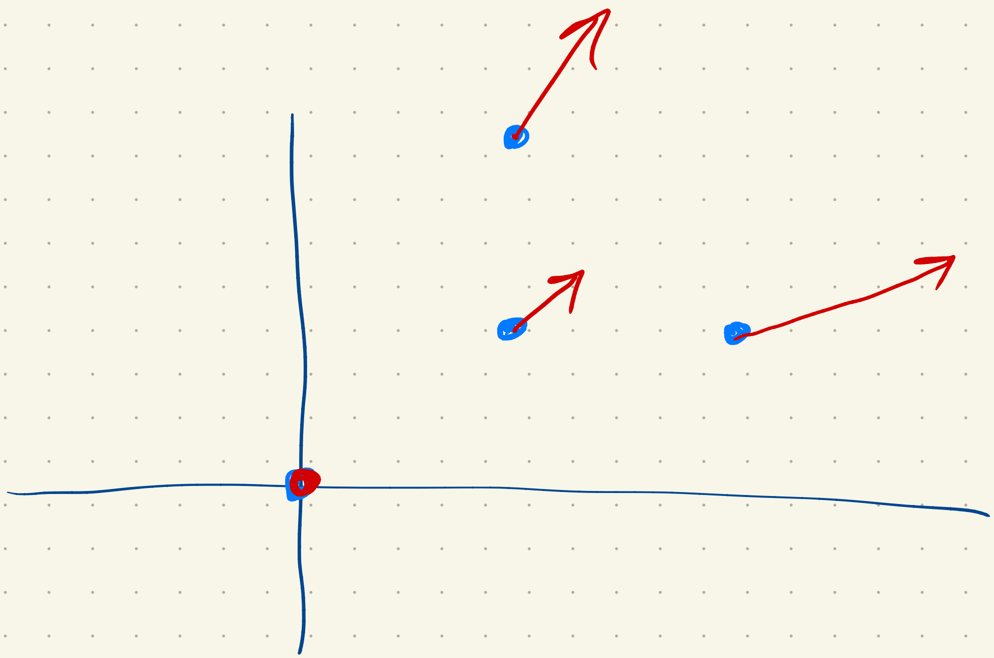
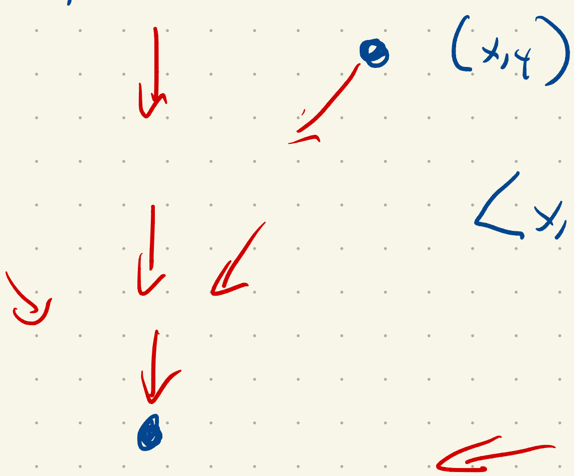
$\vec{F} = \langle x^2, xy \rangle$



P	$\vec{F}$
$(0,0)$	$\langle 0, 0 \rangle$
$(1,1)$	$\langle 1, 1 \rangle$
$(1,2)$	$\langle 1, 2 \rangle$
$(2,1)$	$\langle 4, 2 \rangle$

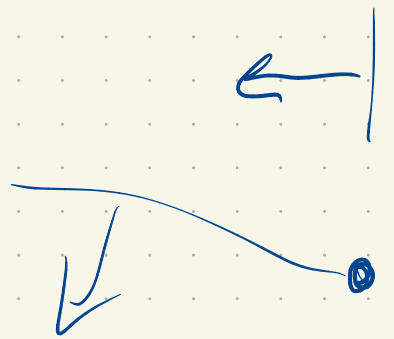
$\langle x, y \rangle$



$\langle x, y \rangle$

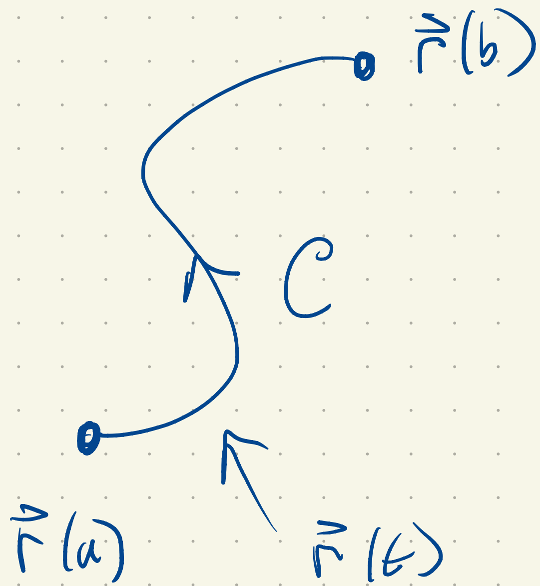
$\langle x, y \rangle$

$$-\left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \right\rangle$$



FTC

$$\int_a^b f'(t)\,dt = f(b) - f(a)$$



$$\vec{F} = \vec{\nabla} f$$

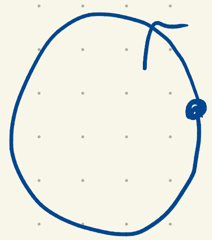
Fund Thm of Line Integrals

$$\boxed{\int_C \vec{\nabla} f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))}$$

$\vec{\nabla} T \cdot \vec{r}\,'$

$\vec{r}(t) = \langle x(t), y(t) \rangle \quad a \le t \le b$

$\vec{r}\,'(t) = \langle x'(t), y'(t) \rangle$



$$\int_C \vec{\nabla} f \cdot d\vec{r} = \int_a^b \nabla f \cdot \vec{r}\,'(t)\, dt$$

$$= \int_a^b \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle \cdot \langle x', y' \rangle\, dt$$

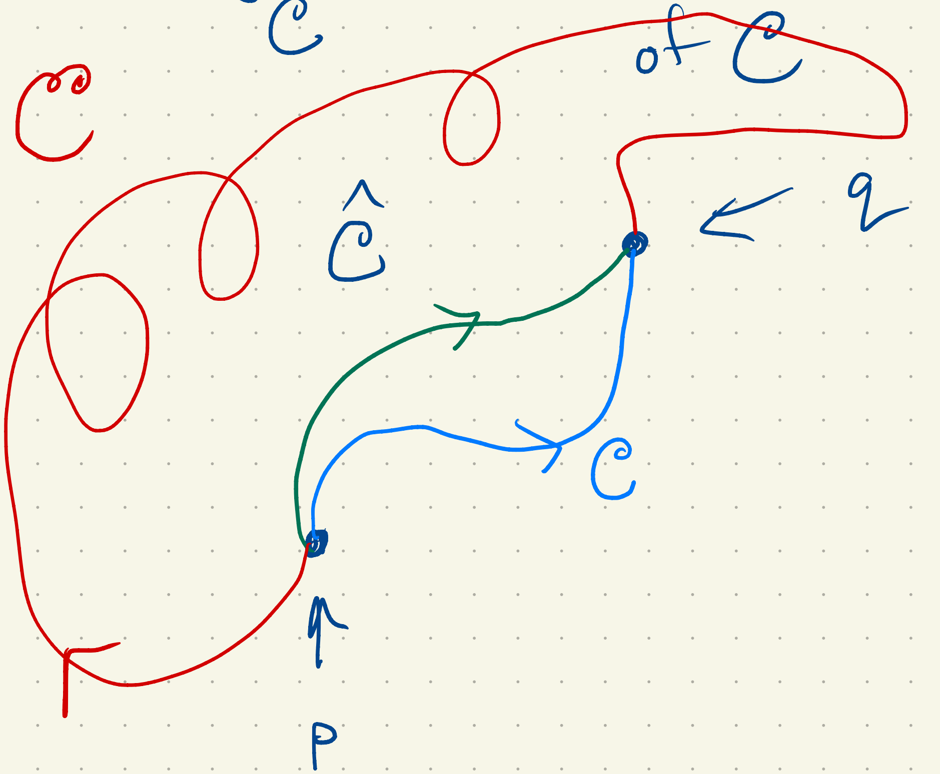
$$= \int_a^b \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}\, dt$$

$$= \int_a^b \frac{d}{dt} f(\vec{r}(t))\, dt$$

$$= f(\vec{r}(b)) - f(\vec{r}(a))$$

$\underbrace{\vec{F}}_{\text{conservative vector field}} = \vec{\nabla} \underbrace{f}_{\text{potential for } \vec{F}}$

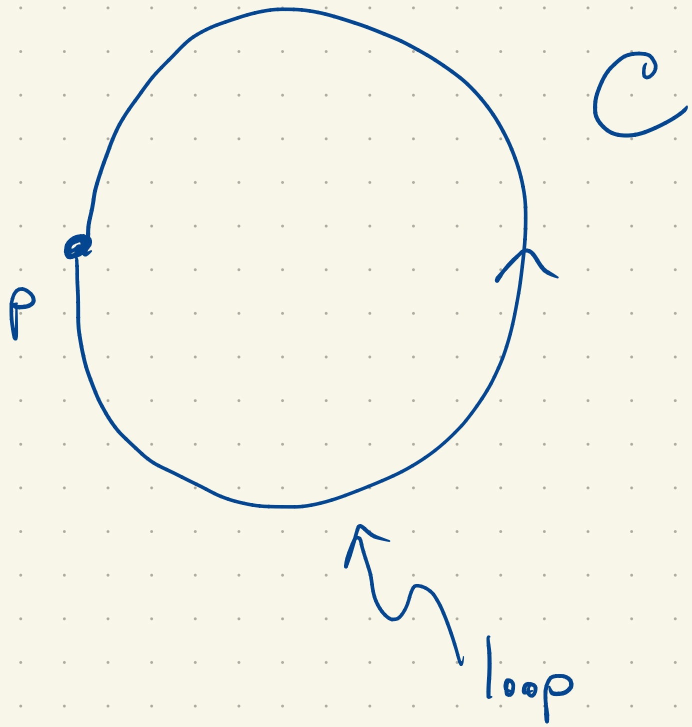
$\int_C \vec{\nabla} f \cdot d\vec{r}$ depends only on the endpoints (with orientation) of C



$$\int_C \vec{\nabla} f \cdot d\vec{r} = f(q) - f(p)$$

$$\int_{\hat{C}} \vec{\nabla} f \cdot d\vec{r} = f(q) - f(p)$$

$$\int_{\mathcal{C}} \vec{\nabla} f \cdot d\vec{r} = f(q) - f(p)$$



$$\int_C \vec{\nabla} f \cdot d\vec{r} = f(p) - f(p)$$
$$= 0$$

Three properties of conservative vector fields $\vec{F} = \langle P, Q \rangle$

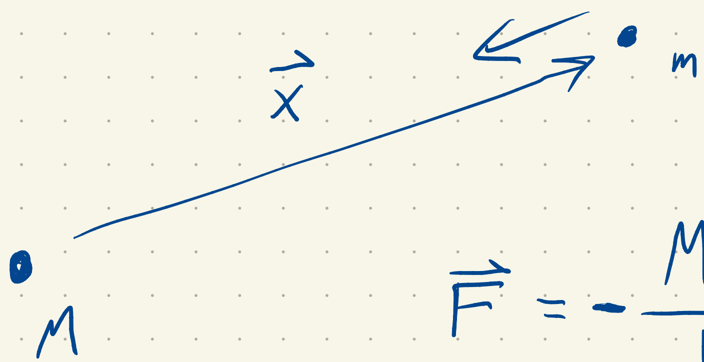
a) the integral over a curve depends only on the end points

b) the integral around a loop is zero

c) $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$, $\dfrac{\partial P}{\partial z} = \dfrac{\partial R}{\partial x}$, $\dfrac{\partial Q}{\partial z} = \dfrac{\partial R}{\partial y}$ $\langle P, Q, R \rangle$

$$\vec{F} = \langle P, Q \rangle = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

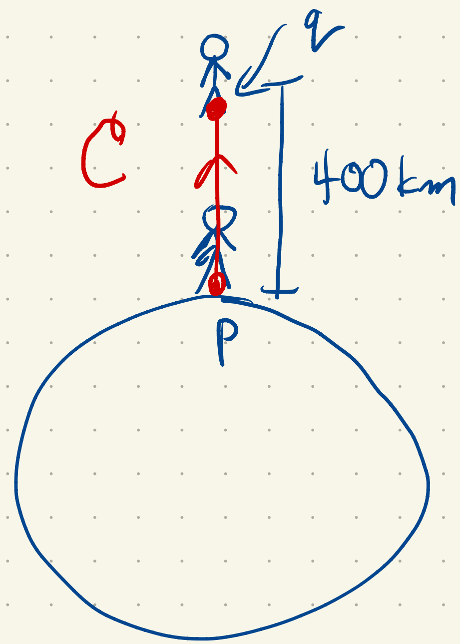
$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y} = \frac{\partial}{\partial x} Q = \frac{\partial Q}{\partial x}$$



$$\vec{F} = -\frac{MmG}{|\vec{x}|^2}\frac{\vec{x}}{|\vec{x}|} = -\frac{MmG}{|\vec{x}|^3}\vec{x}$$

Claim: $\vec{F} = \nabla f$

$$f = \frac{MmG}{|\vec{x}|}$$



work $\int_C \vec{F} \cdot d\vec{r} = f(q) - f(p)$

$= MmG\left(\frac{1}{|\vec{x}(q)|} - \frac{1}{|\vec{x}(p)|}\right)$

$|\vec{x}(p)| = R_e$ radius of earth

$= 6.365 \times 10^6$ m

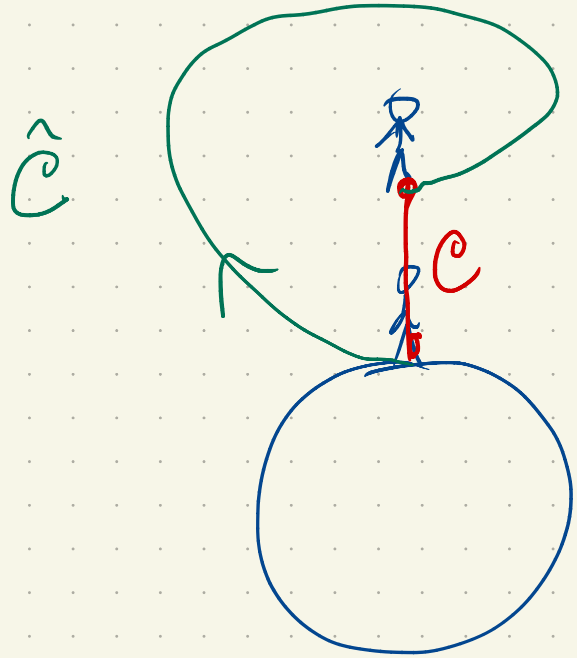
$|\vec{x}(q)| = R_o$ radius in orbit

$= 6.765 \times 10^6$ m

$MG = 3.98 \times 10^{14}$

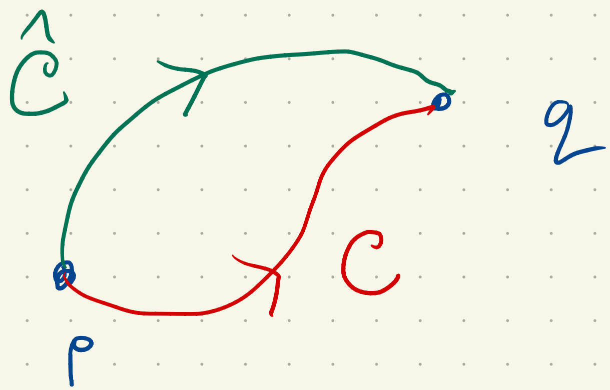
$m = 80$ kg

$E = -296$ MJ



$\int_C \vec{F} \cdot d\vec{r}$ depends only on endpoints

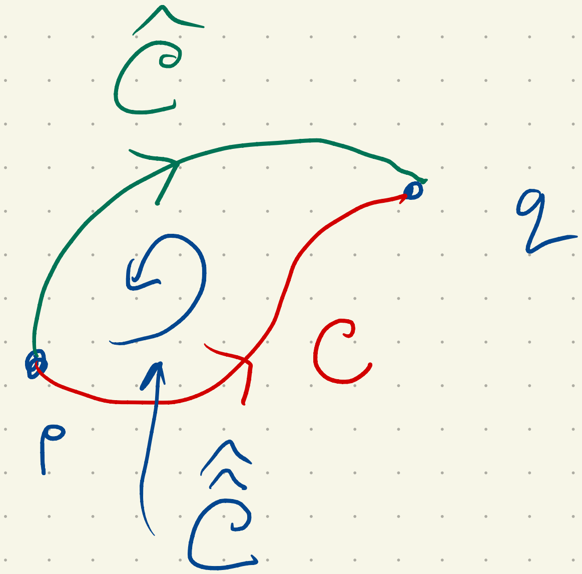
$\vec{F}$ Suppose $\oint_C \vec{F} \cdot d\vec{r} = 0$ whenever C is a loop



Want to show

$\int_C \vec{F} \cdot d\vec{r}$ depends only on endpoints

$$\int_C \vec{F}\cdot d\vec{r} \overset{?}{=} \int_{\hat{C}} \vec{F}\cdot d\vec{r}$$

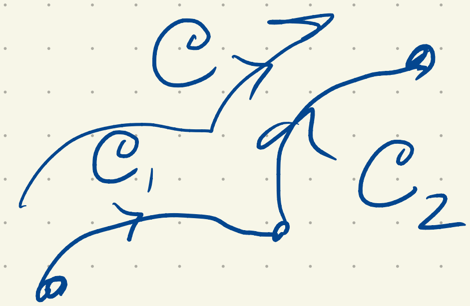


$$\int_C \vec{F}\cdot d\vec{r} - \int_{\hat{C}} \vec{F}\cdot d\vec{r} \overset{?}{=} 0$$

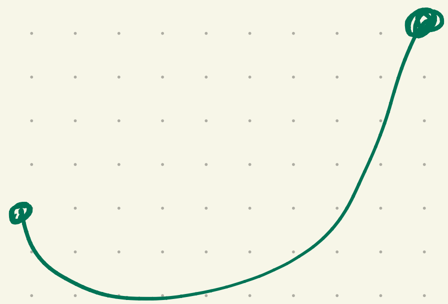
$$\oint_{\hat{\hat{C}}} \vec{F}\cdot d\vec{r} = 0$$

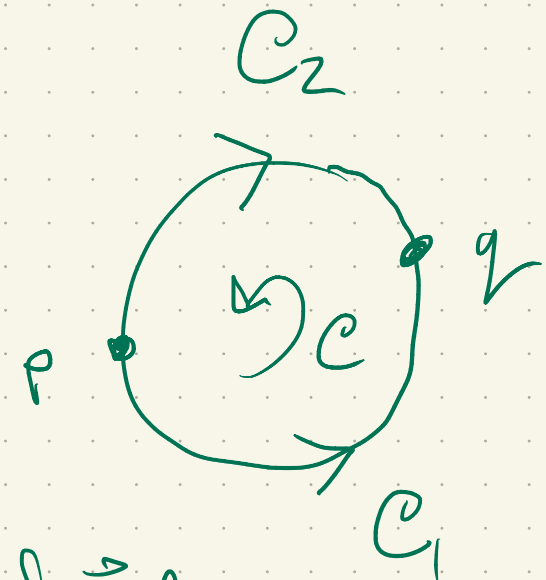
$$\|$$

$$\int_C \vec{F}\cdot d\vec{r} - \int_{C_2} \vec{F}\cdot d\vec{r} = 0$$



$$\int_C \vec{F}\cdot d\vec{r} = \int_{C_1} \vec{F}\cdot d\vec{r} + \int_{C_2} \vec{F}\cdot d\vec{r}$$





$$\int_C \vec{F}\cdot d\vec{r} = \int_{C_1} \vec{F}\cdot d\vec{r} - \int_{C_2} \vec{F}\cdot d\vec{r}$$

$$= 0.$$

$$\vec{F}(x,y) = \langle \underbrace{3+2xy}_{P}, \underbrace{x^2-3y^2}_{Q} \rangle$$

$$\frac{\partial P}{\partial y} = 2x \qquad \frac{\partial Q}{\partial x} = 2x$$

same.

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$ Is $\vec{F}$ conservative?

f

$$\boxed{\frac{\partial f}{\partial x} = 3 + 2xy}$$

$$\frac{\partial f}{\partial y} = x^2 - 3y^2$$

$$\frac{\partial f}{\partial x} = 3 + 2xy$$

$$f = 3x + x^2 y + h(y)$$

$$\frac{\partial f}{\partial y} = 0 + x^2 + h'(y)$$

$h'(y) = -3y^2$ $h(y) = -y^3$

$$f(x,y) = 3x + x^2y - y^3$$