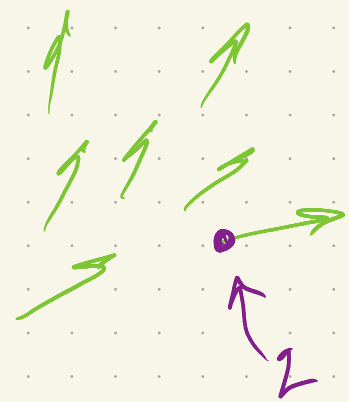
$\vec{E}$ electric field

Particle with charge q experiences a force

$q\vec{E}$



force N

$[q] = C$

$[\vec{E}]$

$[q\vec{E}] = N$

$[q][\vec{E}] = N$

$$[\vec{E}] = N/C$$

$$= \frac{kg\ m}{s^2}\frac{1}{C}$$

$$= \frac{kg\ m^2}{s^2}\frac{1}{C}\frac{1}{m}$$

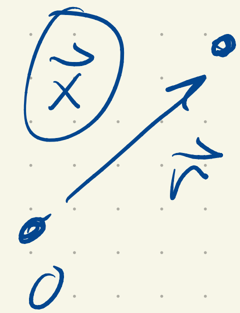
$$= \frac{J}{C}\,\frac{1}{m}$$

volt

$$= V/m$$

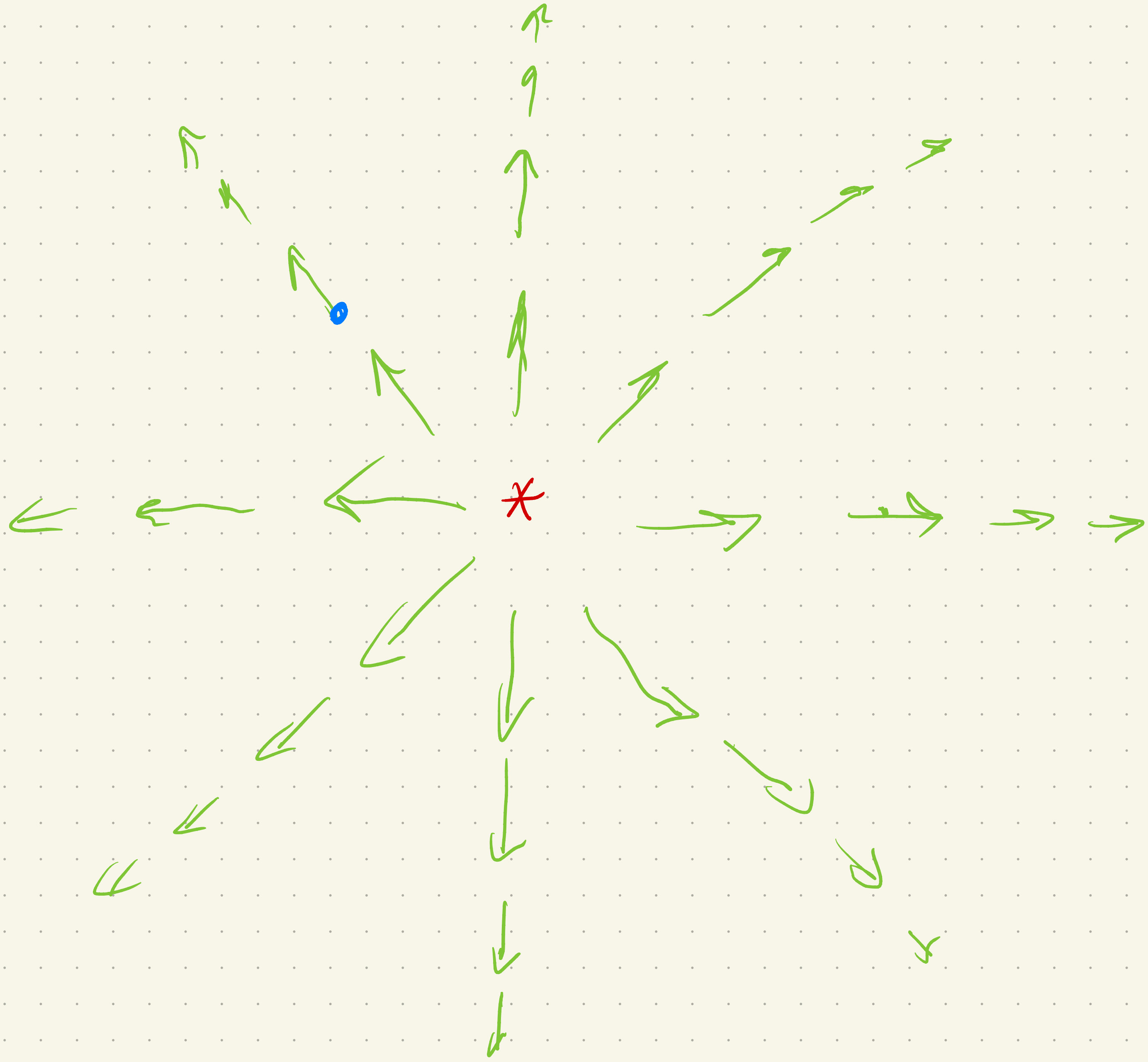
a stationary point particle at the origin with electric charge Q

generates an electric field



$$\vec{E} = Q\,\frac{\vec{x}}{|\vec{x}|^3} = Q\,\frac{\vec{r}}{|\vec{r}|^3}$$

$$= \frac{Q}{|\vec{x}|^2}\,\frac{\vec{x}}{|\vec{x}|}$$



3C

$$E = \langle -1, 2, 3 \rangle$$

$$\vec{F} = 3 \cdot \langle -1, 2, 3 \rangle = \langle -3, 6, 9 \rangle \, N$$

$$\vec{E} = \frac{Q}{|\vec{x}|^2} \frac{\vec{x}}{|\vec{x}|} \longleftarrow$$

is a conservative vector field

this is a gradient

$$f(x,y,z) = -Q\,(x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial f}{\partial x} = -Q\left(\frac{-1}{2}\right)(x^2+y^2+z^2)^{-3/2}\, 2x$$

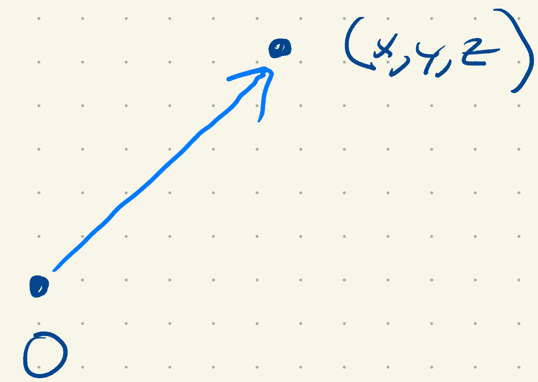
$$= Q\left(x^2+y^2+z^2\right)^{-3/2} x$$

$$\frac{\partial f}{\partial y} = Q\left(x^2+y^2+z^2\right)^{-3/2} y$$

$$\frac{\partial f}{\partial z} = Q\left(x^2+y^2+z^2\right)^{-3/2} z$$

$$\vec{x} = \langle x, y, z \rangle$$

$$\vec{\nabla} f = Q\left(x^2+y^2+z^2\right)^{-3/2} \langle x, y, z \rangle$$



$$= Q\left(x^2+y^2+z^2\right)^{-3/2} \vec{x}$$

$$= Q\left(\|\vec{x}\|^2\right)^{-3/2} \vec{x}$$

$$\vec{x}$$

$$= Q\,\|\vec{x}\|^{-3}\,\vec{x}$$

$$= \frac{Q\vec{x}}{\|\vec{x}\|^3} = \frac{Q}{\|\vec{x}\|^2}\frac{\vec{x}}{\|\vec{x}\|} = \vec{E}\ !$$

x

$$\vec{V} = \langle a(x,y), b(x,y) \rangle$$

If $\vec{V} = \vec{\nabla} f$ then $\vec{V} = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$

$$\frac{\partial}{\partial y} \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \frac{\partial f}{\partial y}$$

$\vec{V} = \langle y, -3x \rangle$ is it conservative?

No: $\frac{\partial}{\partial y} y$ vs. $\frac{\partial}{\partial x}(-3x)$

1 vs. -3 $\qquad 1 \neq -3$ so

$\vec{V}$ is not conservative.

$$\vec{V} = \langle a, b, c \rangle$$

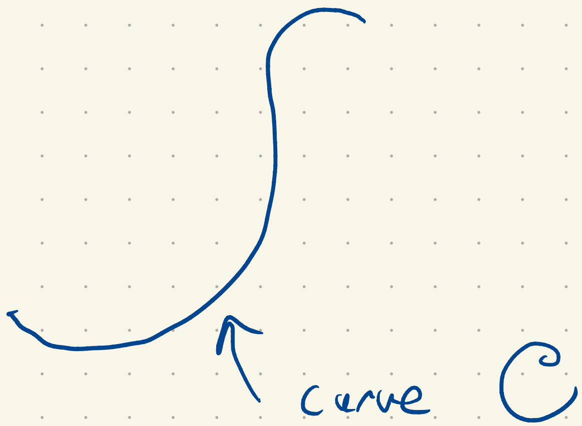
$$\left.\begin{aligned} \frac{\partial a}{\partial y} &= \frac{\partial b}{\partial x} \\ \frac{\partial b}{\partial z} &= \frac{\partial c}{\partial y} \\ \frac{\partial a}{\partial z} &= \frac{\partial c}{\partial x} \end{aligned}\right] \rightarrow \text{necessary for } \vec{V} \text{ to be conservative}$$

$$\frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y}$$

$$\frac{\partial}{\partial z}\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\frac{\partial f}{\partial z}$$

$$\frac{\partial}{\partial z}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\frac{\partial f}{\partial z}$$

Line Integrals

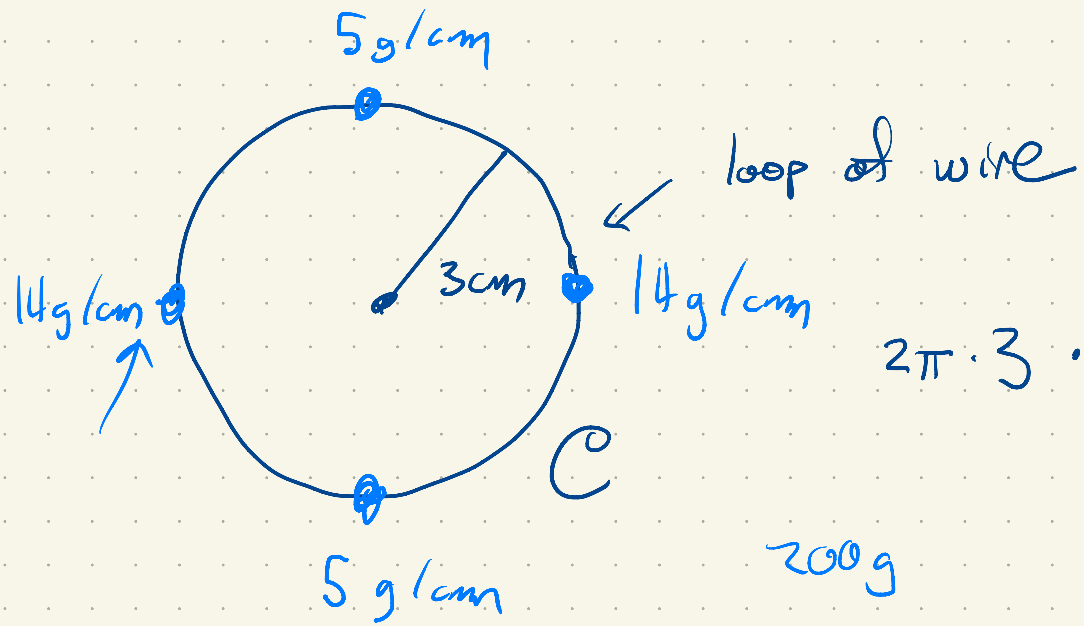


How to integrate "things" along curves

Two types of things

a) "desity" type

b) "work" type



(uniform density ρ)

$\hookrightarrow$ 2 g/cm

$$2\pi \cdot 3 \cdot 2 = 12\pi \text{ g}$$

200g

$\rho = 5 + x^2$ g/cm

$$\boxed{\int_C \rho(x,y)\, \underbrace{ds}_{\text{"arclength"}}} = \text{mass of the loop}$$

Steps: 1) parameterize the curve

$\vec{r}(t) = \langle 3\cos t, 3\sin t \rangle \quad 0 \leq t \leq 2\pi$

2) rewrite the integrand $\rho(x,y) = 5 + x^2$ in terms of the parameter t.

$x(t) = 3\cos t$

$y(t) = 3\sin t$

$$\rho = 5 + (3\cos(t))^2 = 5 + 9\cos^2 t$$

3) Compute $\|\vec{r}\,'(t)\|$ $\quad \vec{r}\,'(t) = 3\langle -\sin t, \cos(t) \rangle$

$\|\vec{r}\,'(t)\| = 3$

4) $$\int_C \rho(x,y)\,ds = \int_0^{2\pi} (5 + 9\cos^2 t)\,\|\vec{r}\,'(t)\|\,dt$$

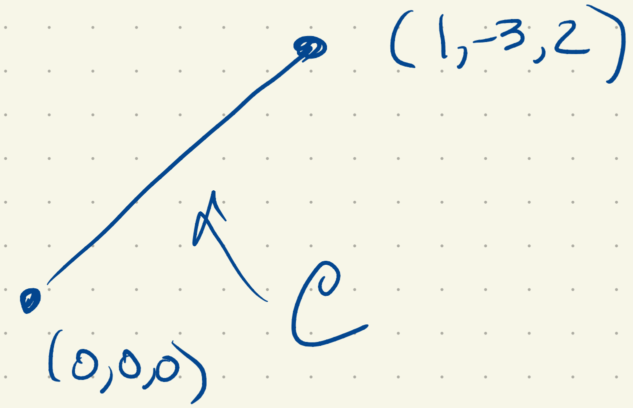
$$= \int_0^{2\pi} (5 + 9\cos^2 t)\,3\,dt$$

$$= 3\left(10\pi + 9\int_0^{2\pi} \cos^2(t)\,dt\right)$$

$$= 3\left(10\pi + 9\,\frac{1}{2}(2\pi)\right)$$

$$= 30\pi + 27\pi$$

$$= 57\pi \text{ g}$$



$$\int_C x+y^2-2z\,ds$$

$$\vec{r}(t) = \langle t, -3t, 2t \rangle \qquad 0 \le t \le 1$$

$$\vec{r}\,'(t) = \langle 1, -3, 2 \rangle$$

$$\|\vec{r}\,'(t)\| = (1^2+3^2+2^2)^{1/2} = \sqrt{14}$$

$$\int_C x+y^2-2z\,ds = \int_0^1 \left(t + (-3t)^2 - 2(2t)\right) \overbrace{\|\vec{r}\,'(t)\|}^{\text{scaling factor}}\,dt$$

$$= \int_0^1 (t + 9t^2 - 4t)\sqrt{14}\,dt$$

$$= \sqrt{14} \int_0^1 -3t + 9t^2 \, dt$$

$$= \frac{3}{2}\sqrt{14}$$