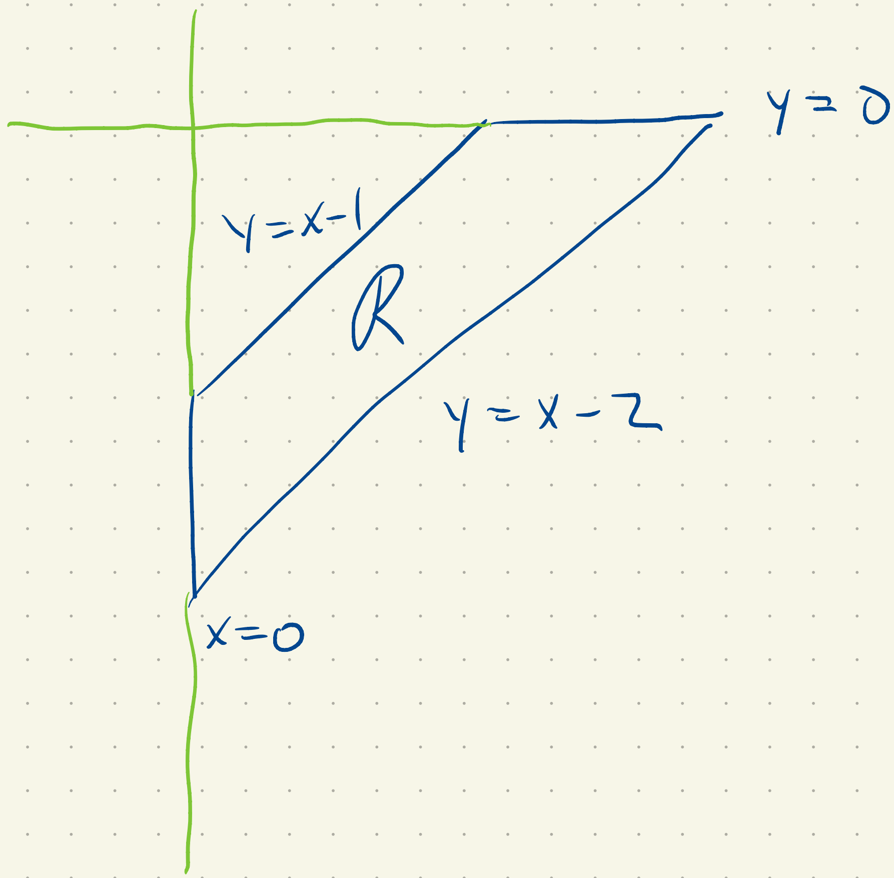
$= r\left(\cos^2\theta + \sin^2\theta\right)$

$= r$



$$\iint_R e^{\frac{x+y}{x-y}}\, dA$$

$u = x + y$
$v = x - y$

$u + v = 2x \Rightarrow x = \frac{u+v}{2}$

$u - v = 2y \Rightarrow y = \frac{u-v}{2}$

$x = \frac{u+v}{2}$

$y = \frac{u-v}{2}$

$$\iint_S e^{u/v} \bigcirc \, du\, dv$$

$1/2$

$$\frac{\partial x}{\partial u} = \frac{1}{2} \qquad \frac{\partial x}{\partial v} = \frac{1}{2}$$

$$\frac{\partial y}{\partial u} = \frac{1}{2} \qquad \frac{\partial y}{\partial v} = -\frac{1}{2}$$

$$J = \begin{vmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{vmatrix}$$

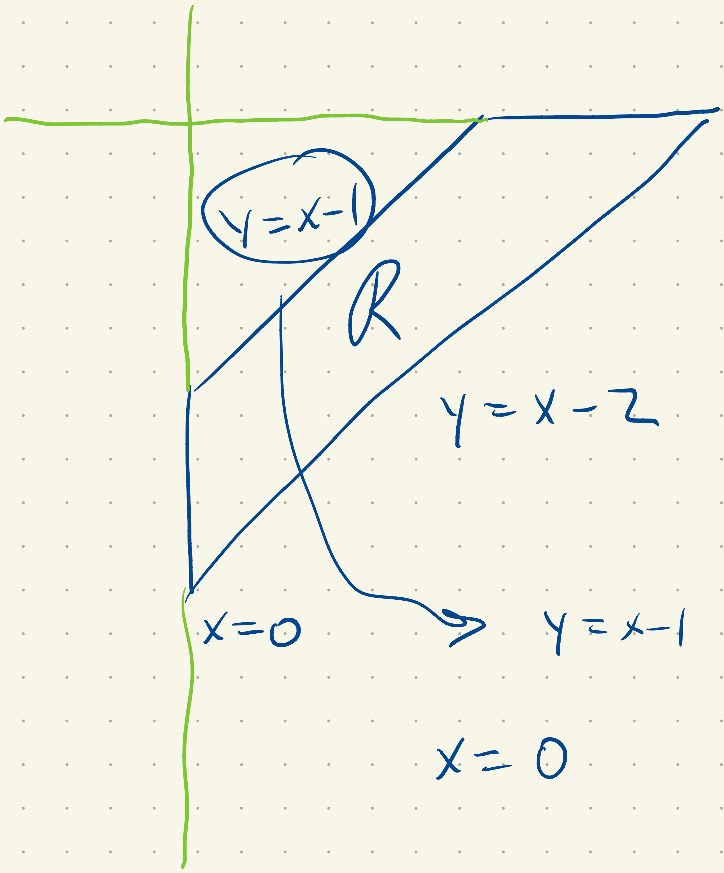
$$= \frac{1}{2}\cdot\left(-\frac{1}{2}\right) - \frac{1}{2}\cdot\frac{1}{2}$$

$$= -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

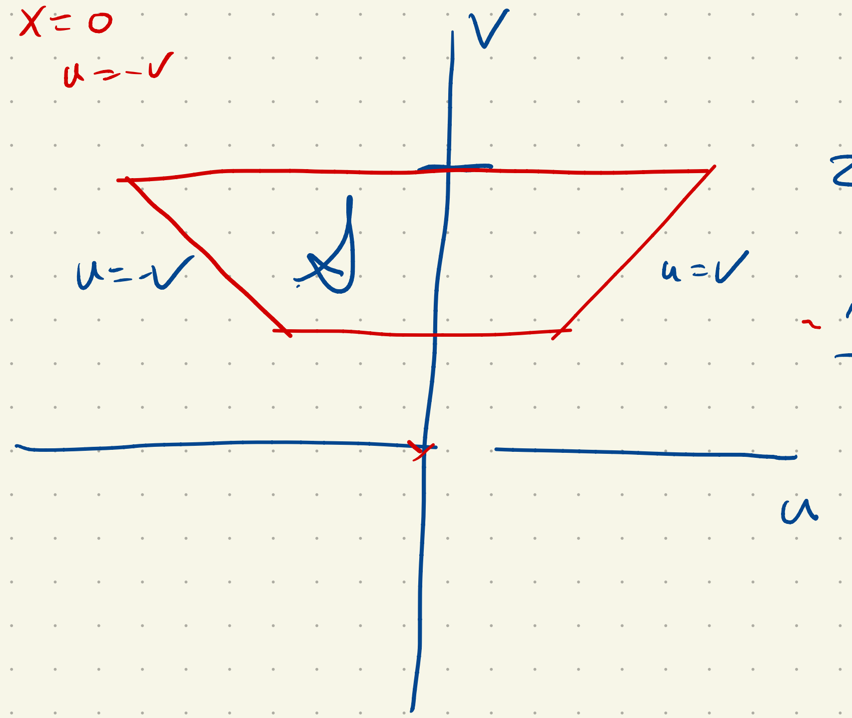
$$\boxed{J = 1/2}$$

$$\iint_S e^{u/v} \, \frac{1}{2} \, du\, dv$$

$u = x+y$
$v = x-y$



$y=0$



$y-x=-2$

$x-y=2$

$v=2$

$y=x-1 = 1 = x-y = v$

$x=0 \qquad y=0$

$\frac{u+v}{2}=0 \qquad \frac{u-v}{2}=0$

$u=-v \qquad u=v$

$$\int_1^2 \int_{-v}^{v} e^{u/v} \, \frac{1}{2} \, du \, dv$$

$$\int_1^2 \int_{-v}^{v} e^{u/v} \frac{1}{2} \, du \, dv = \frac{1}{2} \int_1^2 \int_{-v}^{v} e^{u/v} \, du \, dv$$

$$= \frac{1}{2} \int_1^2 v e^{u/v} \Big|_{u=-v}^{u=+v} dv$$

$$= \frac{1}{2} \int_1^2 v (e^{1} - e^{-1}) \, dv$$

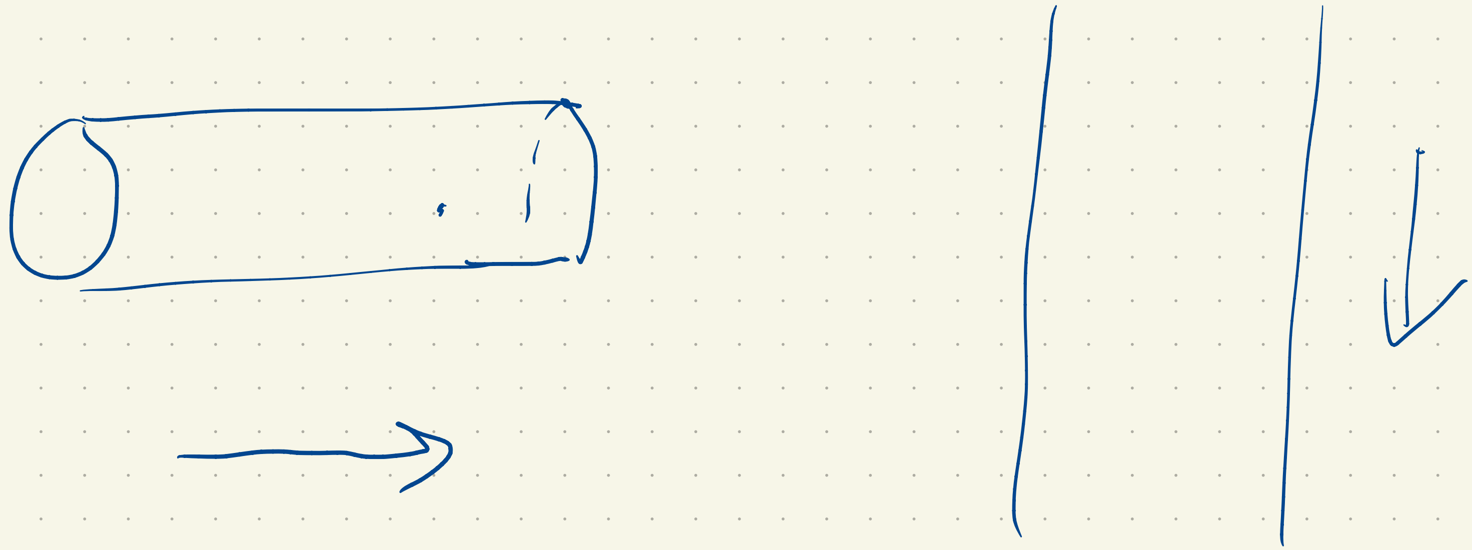
$$= \int_1^2 v \boxed{\left(\frac{e^{1} - e^{-1}}{2} \right)} dv$$

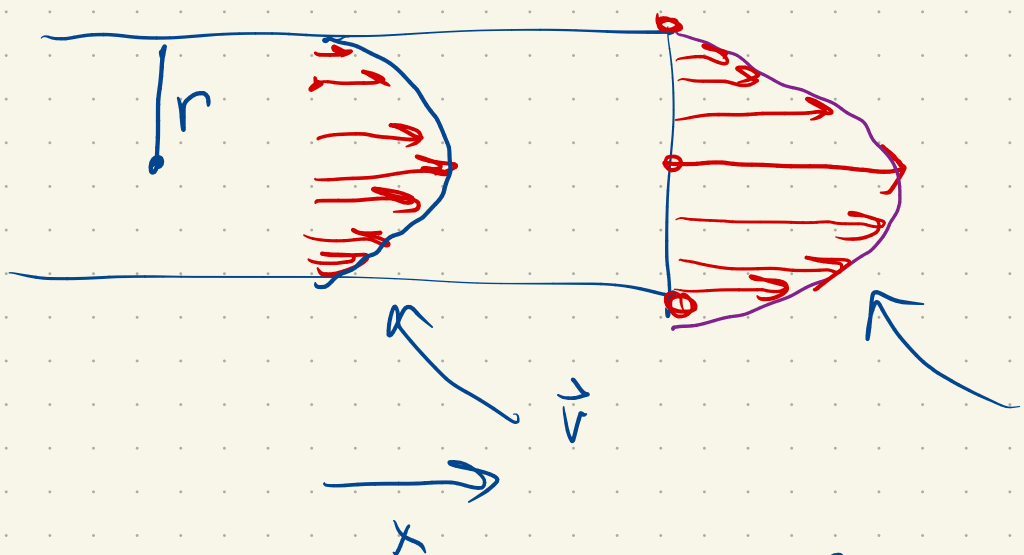
$$\sinh(x) = \frac{e^{x} - e^{-x}}{2}$$

$$= \sinh(1) \int_1^2 v \, dv$$

$$= \sinh(1) \frac{v^2}{2} \Big|_1^2 = \sinh(1) \cdot \frac{3}{2}$$

Vector Fields





Vector field

Poiseille flow

$$\vec{v} = v_0 \left(\frac{r^2 - y^2 - x^2}{r^2} \right)$$

Another example: gradient of a function!

Electric field