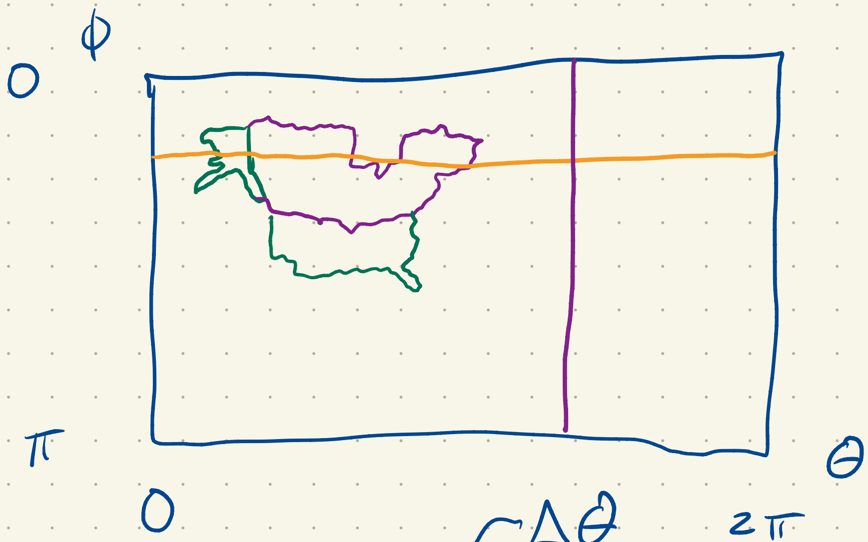
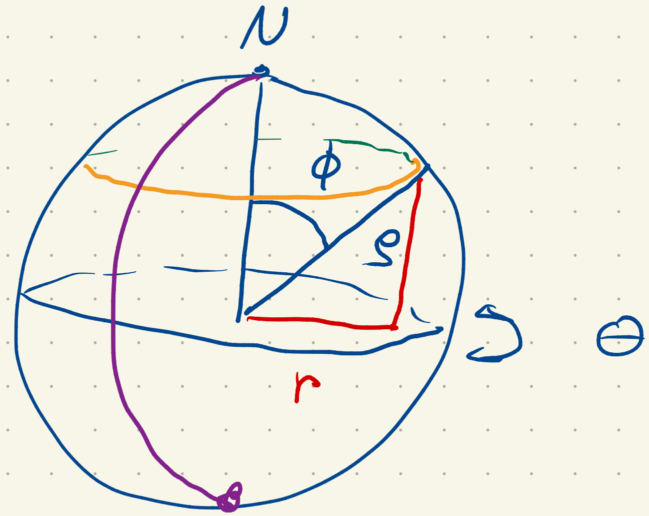
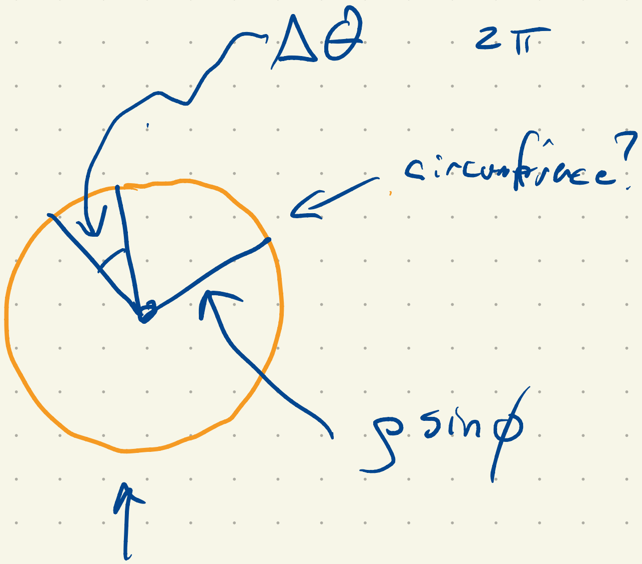
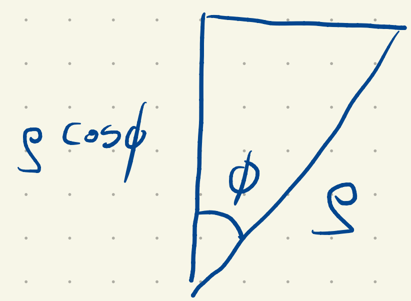


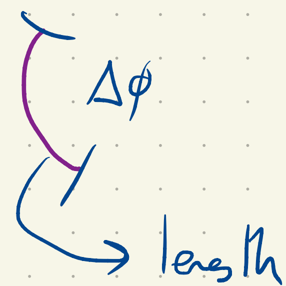




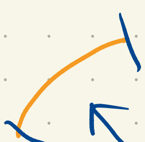
$\rho\sin\phi = r$



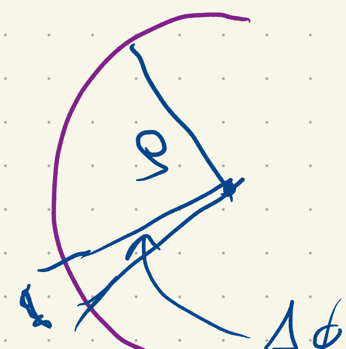
circ $= 2\pi\rho\sin\phi$



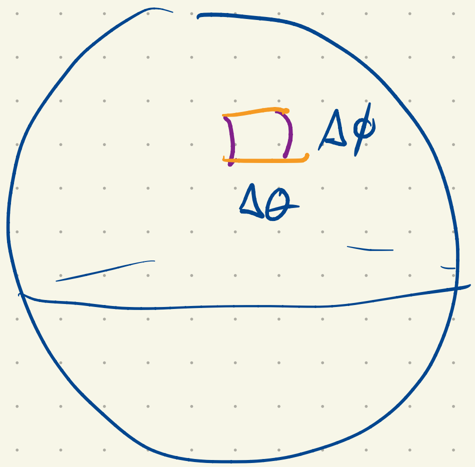
$\pi\rho$



$\Delta\theta\rho\sin\phi$

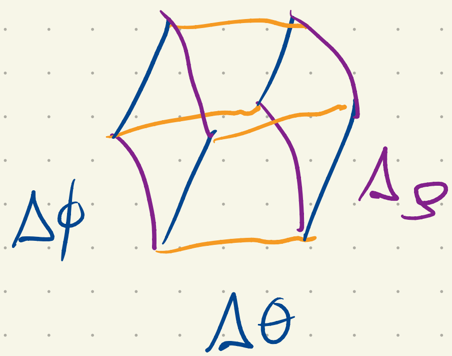


$\Delta\phi\rho$



$$\text{area} \approx (\Delta\theta\, \rho \sin\phi)(\Delta\phi\, \rho)$$

$$= \rho^2 \sin\phi \, \Delta\theta \Delta\phi$$



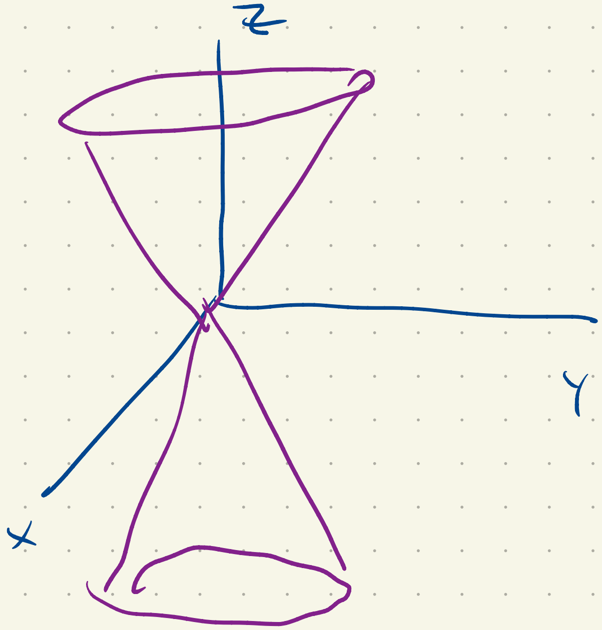
$$\text{Volume} \quad \rho^2 \sin\phi \, \Delta\theta \Delta\phi \Delta\rho$$

$$dV \rightarrow \rho^2 \sin\phi \, d\theta \, d\phi \, d\rho$$

Compute the volume of the region bounded by

$$\rho = 1 \quad \text{and} \quad \rho = 3$$

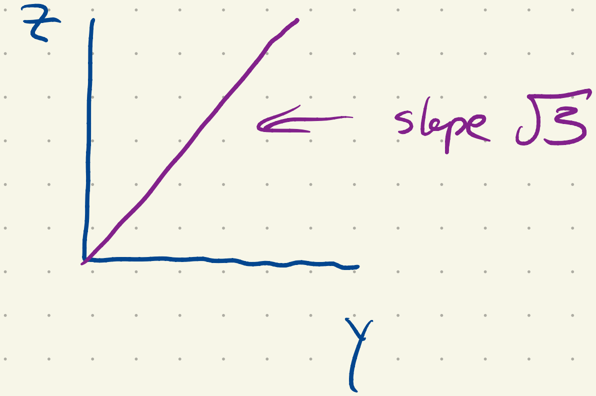
and the cone $z^2 = 3(x^2+y^2)$ and $z > 0$.

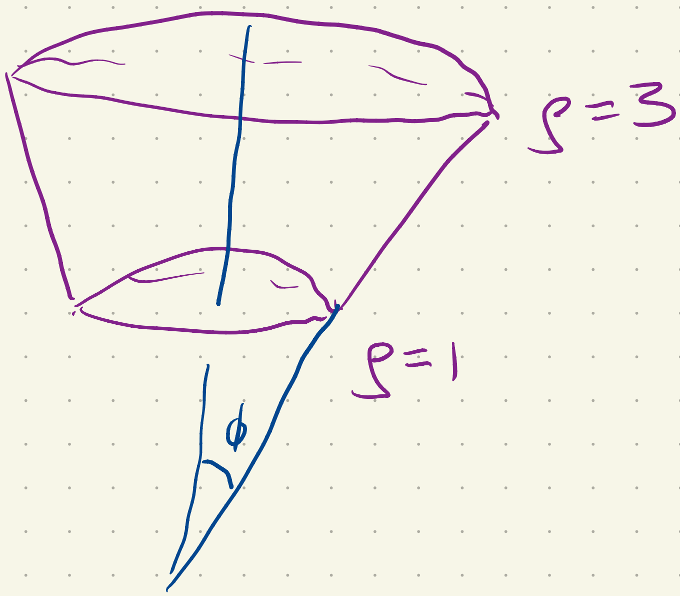


$$z = \pm\sqrt{3}\,(x^2+y^2)^{1/2}$$

$$z = \sqrt{3}\,(y^2)^{1/2}$$

$$z = \sqrt{3}\,|y|$$

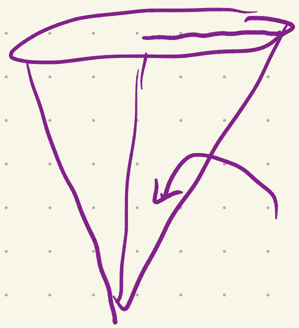




$$0 \le \theta \le 2\pi$$

$$1 \le \rho \le 3$$

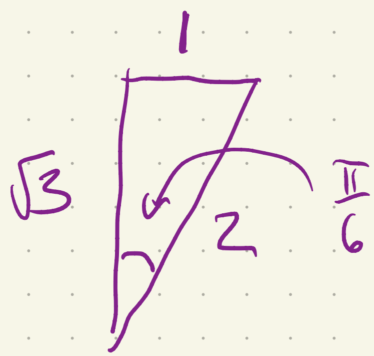
$0 \le \phi \le$? $\frac{\pi}{6}$



$$z^2 = 3(x^2+y^2)$$

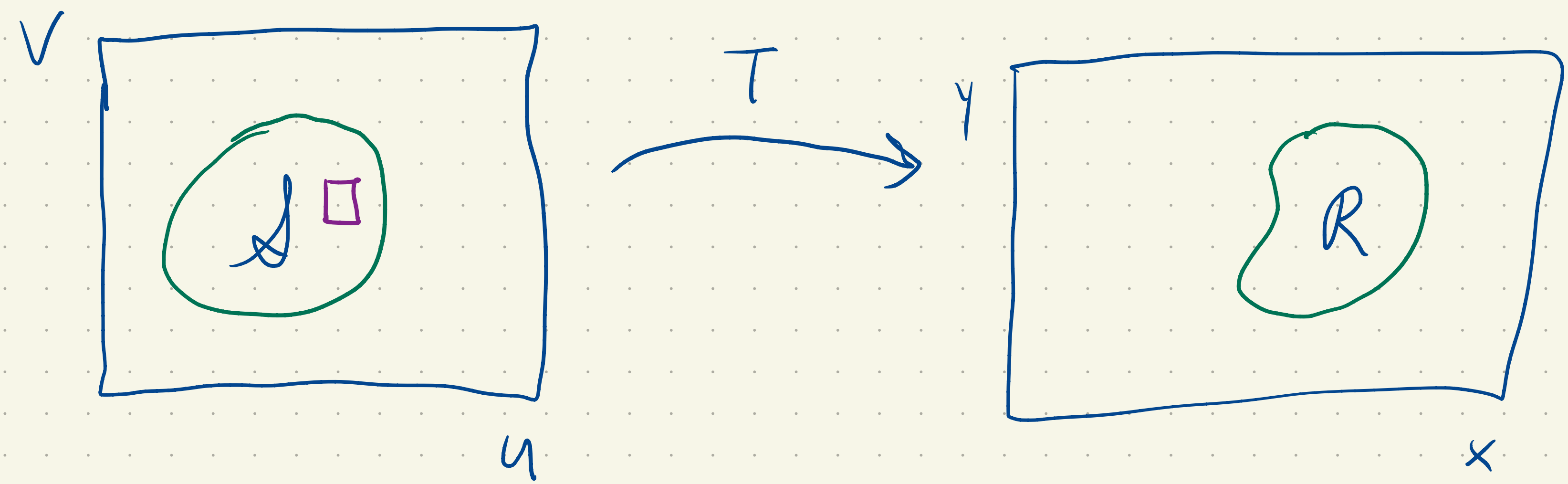
$$z = \sqrt{3}\,(x^2+y^2)^{1/2}$$

$$x=0,\ y=1 \Rightarrow z=\sqrt{3}$$



$$\int_0^{2\pi}\int_1^{3}\int_0^{\pi/6} 1\ \rho^2 \sin\phi\ d\phi\, d\rho\, d\theta = \frac{26\pi}{3}\left(2-\sqrt{3}\right)$$

Changes of coordinates



$$T(u,v) = (x(u,v),\ y(u,v))$$

$$\iint_R f(x,y)\,dA$$

(r,θ) $\quad x(r,\theta) = r\cos\theta$

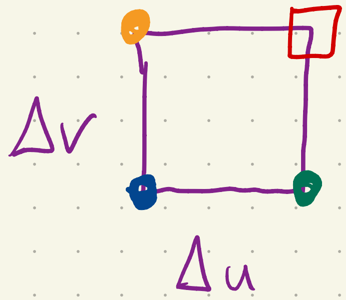
$\quad y(r,\theta) = r\sin\theta$

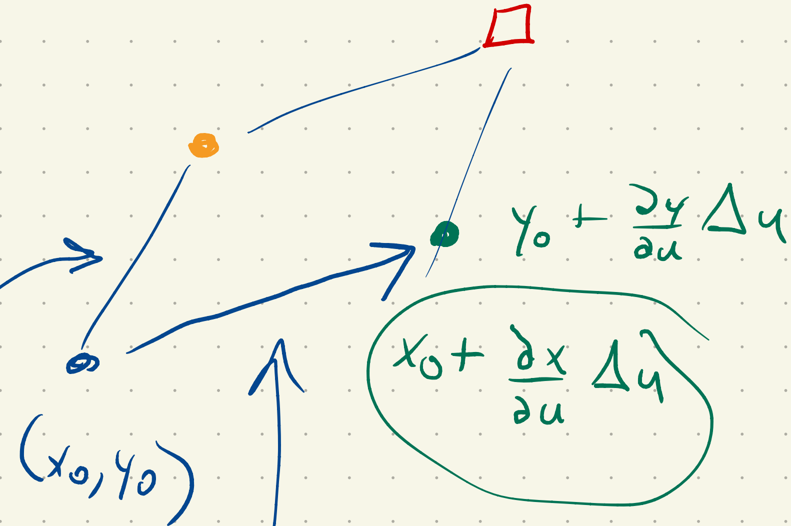
$r \quad dr\,d\theta$

$r \quad dr\,d\theta\,dz$

$\rho^2\sin\phi \quad d\rho\,d\theta\,d\phi$

$$\iint_R f(x,y)\,dA = \iint_{\mathcal{S}} f(x(u,v),\ y(u,v)) \qquad du\,dv$$





What is the "true" (x,y) area of this block.

$$\left\langle \frac{\partial x}{\partial u}, \frac{\partial y}{\partial u} \right\rangle \Delta u$$

$$\left\langle \frac{\partial x}{\partial v}, \frac{\partial y}{\partial v} \right\rangle \Delta v$$

$$\begin{bmatrix} \frac{\partial x}{\partial u}\Delta u & \frac{\partial x}{\partial v}\Delta v \\ \frac{\partial y}{\partial u}\Delta u & \frac{\partial y}{\partial v}\Delta v \end{bmatrix}$$

area of the parallelogram is the det of this matrix

$$\frac{\partial x}{\partial u}\frac{\partial y}{\partial v}\Delta u \Delta v - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}\Delta u \Delta v$$

$$\underbrace{\left(\frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}\right)}\Delta u \Delta v$$

$$\underbrace{\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}}\Delta u \Delta v \longrightarrow \text{det of the } 2\times 2 \text{ block}$$

(absolute value of determinant)

$$\left| \frac{\partial(x,y)}{\partial(u,v)} \right|$$

Jacobian determinant

$$\iint_R f(x,y)\,dA = \iint_S f(x(u,v),\, y(u,v)) \underbrace{\left|\frac{\partial(x,y)}{\partial(u,v)}\right|}_{J} du\,dv$$

$$(r, \theta) \qquad x = r\cos\theta$$

$$y = r\sin\theta$$

$$dA = \underbrace{r}\, dr\, d\theta$$

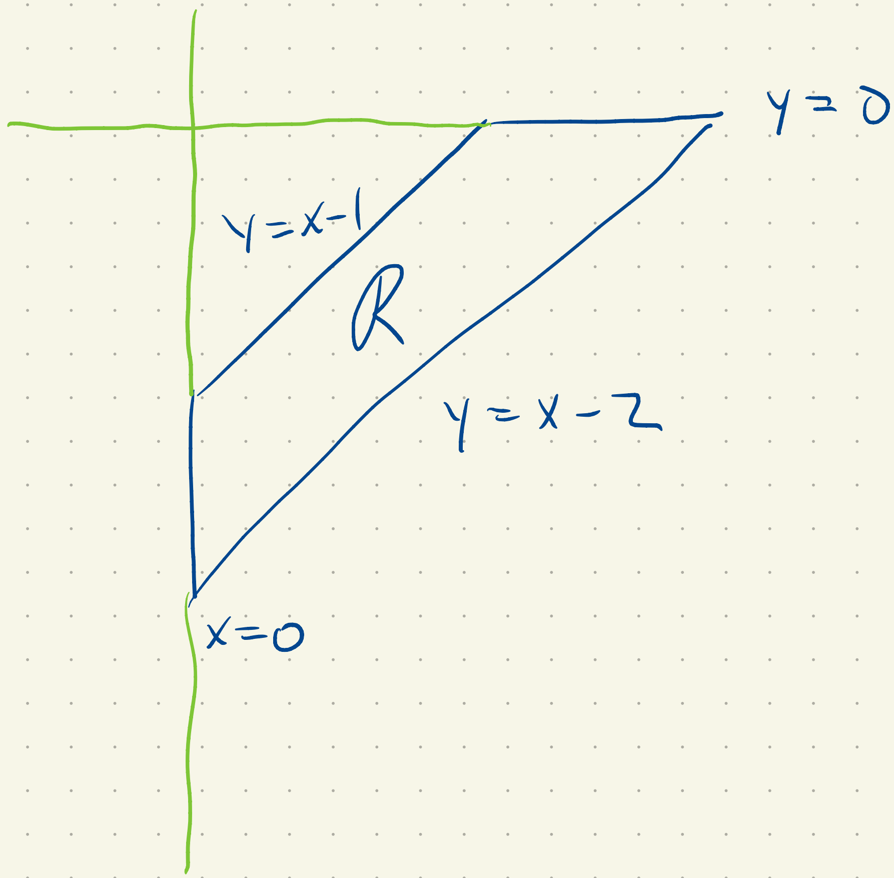
$$\frac{\partial x}{\partial r} = \cos\theta \qquad \frac{\partial x}{\partial \theta} = -r\sin\theta$$

$$\frac{\partial y}{\partial r} = \sin\theta \qquad \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$J = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta$$

$= r(\cos^2\theta + \sin^2\theta)$

$= r$



$$\iint_R e^{\frac{x+y}{x-y}}\, dA$$

$u = x + y$

$v = x - y$

$u + v = 2x \Rightarrow x = \frac{u+v}{2}$

$u - v = 2y \Rightarrow y = \frac{u-v}{2}$

$x = \frac{u+v}{2}$

$y = \frac{u-v}{2}$

$$\iint_{S} e^{u/v} \bigcirc \, du\, dv$$

$$\frac{1}{2}$$

$$\frac{\partial x}{\partial u} = \frac{1}{2} \qquad \frac{\partial x}{\partial v} = \frac{1}{2}$$

$$\frac{\partial y}{\partial u} = \frac{1}{2} \qquad \frac{\partial y}{\partial v} = -\frac{1}{2}$$

$$J = \begin{vmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{vmatrix}$$

$$\iint_{S} e^{u/v} \frac{1}{2} \, du\, dv$$

$$= \frac{1}{2} \cdot \left(-\frac{1}{2}\right) - \frac{1}{2} \cdot \frac{1}{2}$$

$$= -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

$$J = 1/2$$