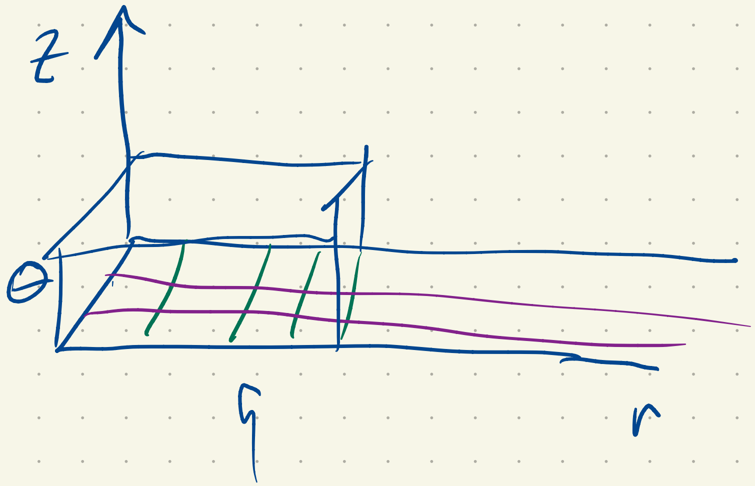
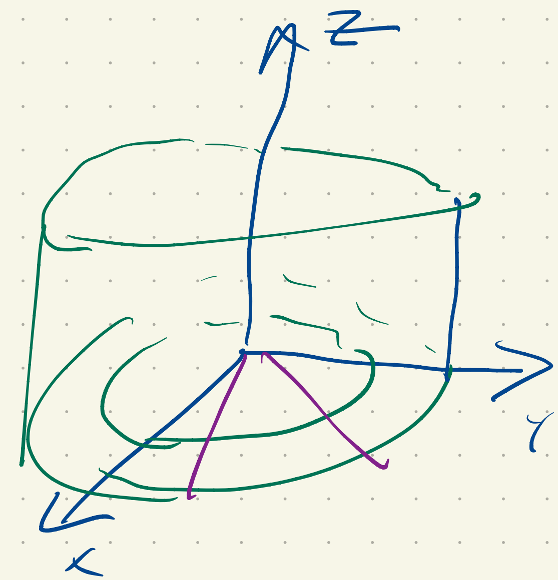
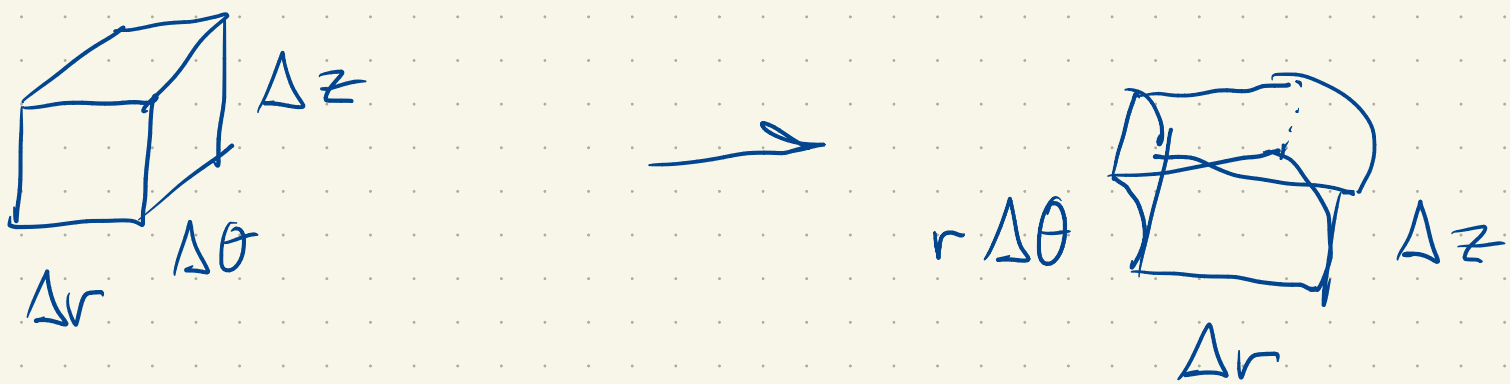
Cylindrical Coordinate





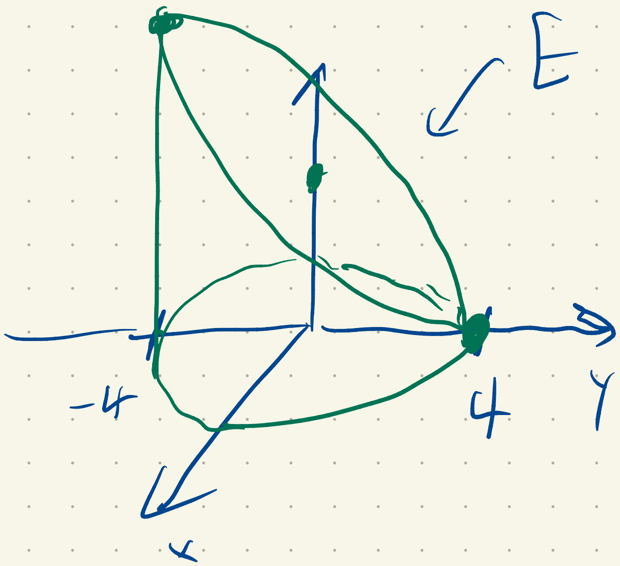


$$r \Delta r \Delta \theta \Delta z$$

Integrate $\sqrt{x^2+y^2}$ over the region bounded by

$$x^2+y^2=16$$
$$z=0$$
$$z=4-y \longrightarrow y=r\sin\theta$$

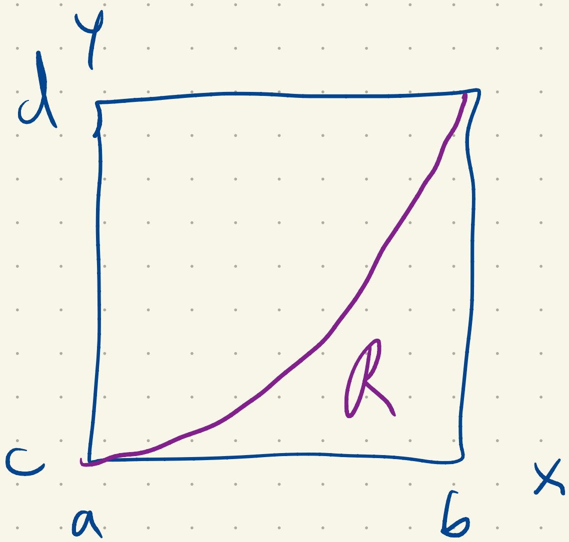
$$\left(x=r\cos\theta \quad z=z \right.$$



$$\iiint_E \sqrt{x^2+y^2}\, dV = \int_0^{2\pi}\int_0^4\int_0^{4-r\sin\theta} r \; dz\, r\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^4\int_0^{4-r\sin\theta} r^2 \, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^4 r^2(4-r\sin\theta)\, dr\, d\theta$$



$$\iint_R f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

$$= \int_a^b \int_c^d f(x,y)\,dy\,dx$$

$$= \int_0^4 \int_0^{2\pi} r^2(4 - r\sin\theta)\,d\theta\,dr$$

$$= \int_0^4 r^2 \int_0^{2\pi} (4 - r\sin\theta)\,d\theta\,dr$$

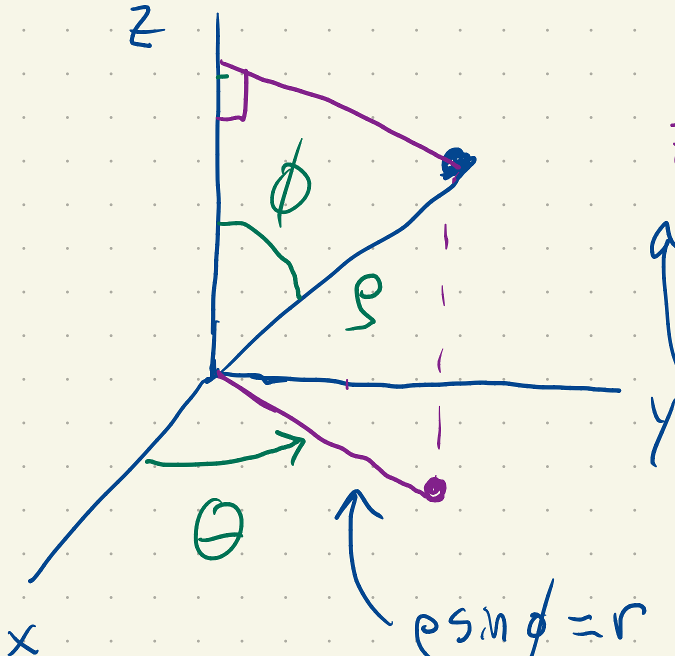
$$= \int_0^4 r^2 \left(4 \cdot 2\pi - 0\right) dr$$

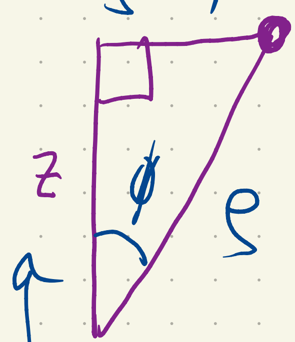
$$= 8\pi \frac{r^3}{3}\Big|_0^4 = \frac{8\pi}{3} 4^3 = \frac{512\pi}{3}$$

Spherical coordinates

(x,y,z) $\quad$ (ρ, θ, ϕ)

$\rho \sin\phi$





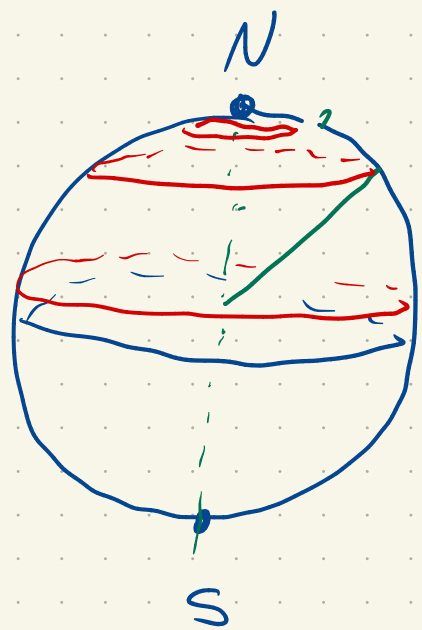
$z = \rho\cos\phi$

θ $\quad$ φ

S $\quad$ S

$0 \leq \varphi \leq \pi$

$\rho\sin\phi = r$



$\rho = \sqrt{x^2+y^2+z}$

$x = r\cos\theta = \rho\sin\phi\cos\theta$

$y = r\sin\theta = \rho\sin\phi\sin\theta$

$$\sqrt{x^2+y^2+z^2} = \rho$$

$$\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi \end{aligned}$$

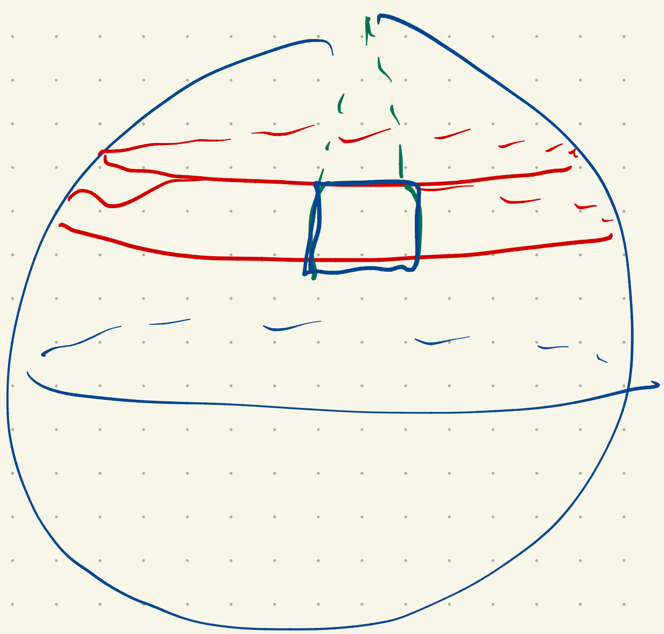
$x = r\cos\theta$

$y = r\sin\theta$

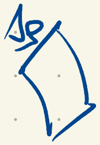
$dA = r\,dr\,d\theta$

$\Delta\theta$ $\Delta\phi$ $\Delta\rho$

$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

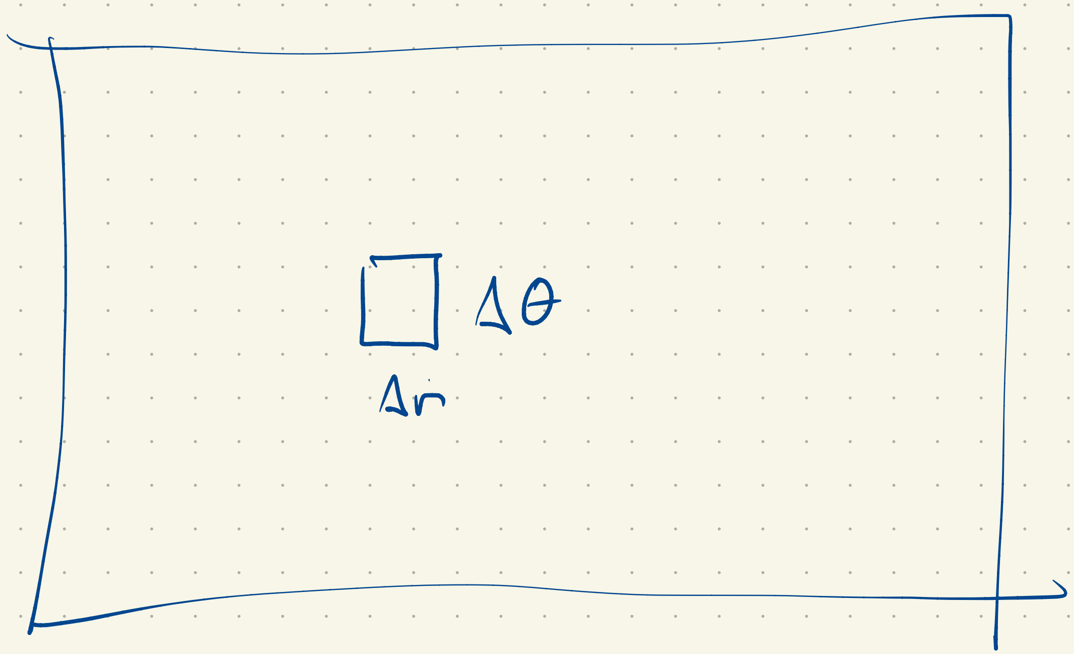


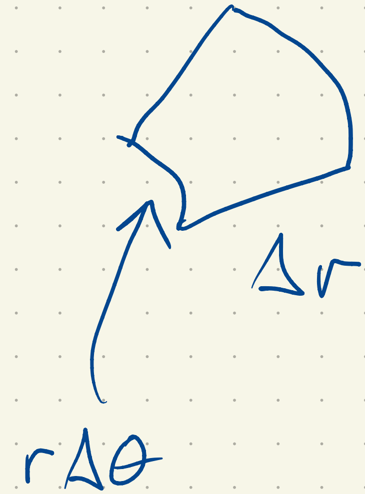
$\sin\phi \, d\phi \, d\theta$

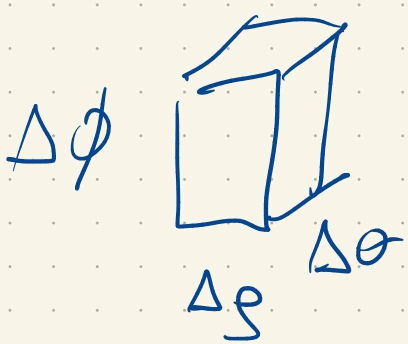


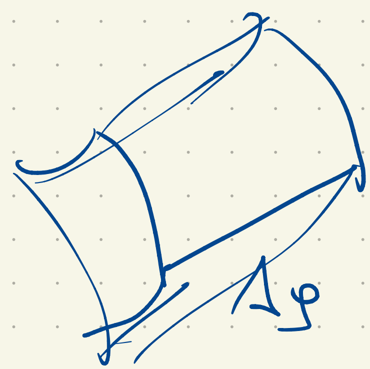
$\Delta\rho$

$\Delta\phi \, \Delta\theta$


Δθ
Δr

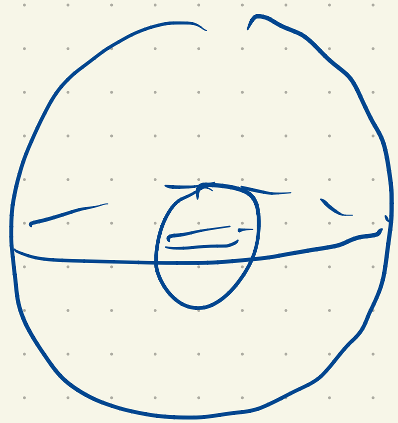

Δr
rΔθ


Δϕ
Δθ
Δρ


Δφ

$0 \le \theta \le 2\pi$

$1 \le \rho \le 2$



E

$z = \rho \cos\phi$

$$\iiint_E z^2 \, dV = \int_0^{2\pi} \int_0^{\pi} \int_1^2 (\rho\cos\phi)^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \frac{124\pi}{15}$$