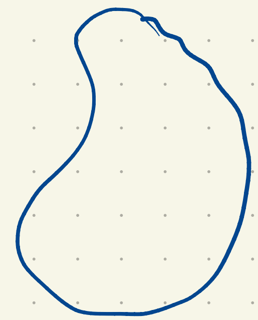
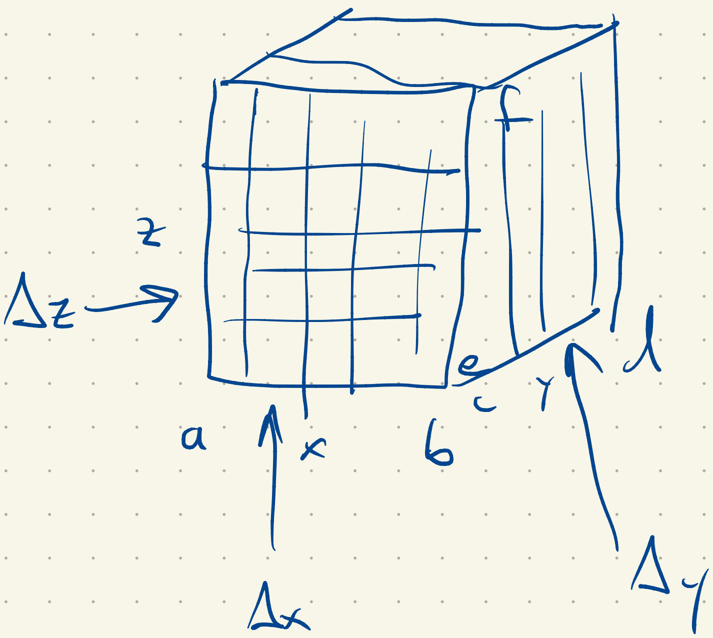
Triple Integrals

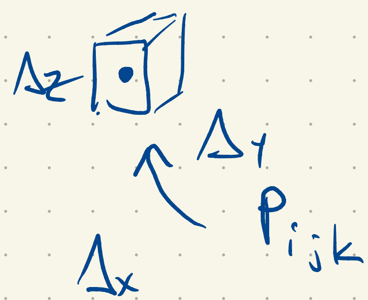
water density $1\, g / cm^3$

$\rho(x,y,z)$





ρ



mass $\approx \rho(P_{ijk})\,\Delta x \Delta y \Delta z$

$$\iiint_E \rho(x,y,z)\, dV$$

volume

We have a Fubini's Theorem

If ρ is continuous then

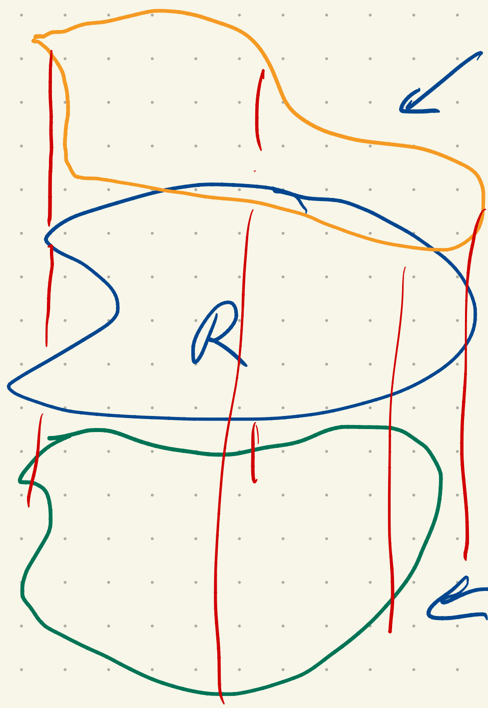
$$\iiint_{E} \rho \, dV = \int_a^b \int_c^d \int_e^f \rho(x,y,z) \, dz \, dy \, dx$$

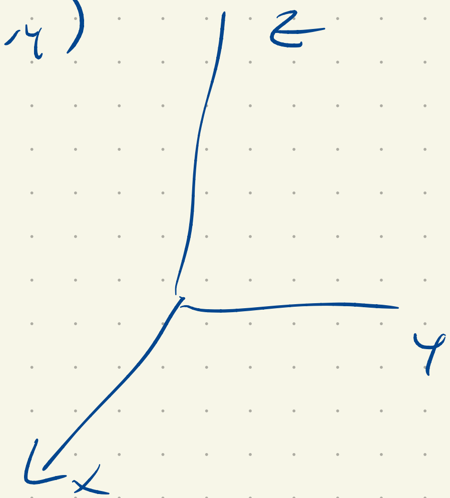
$$= \int_c^d \int_a^b \int_e^f \rho(x,y,z) \, dz \, dx \, dy$$

$= $ etc.

$z = f_2(x,y)$

$\mathcal{E} \rightarrow$





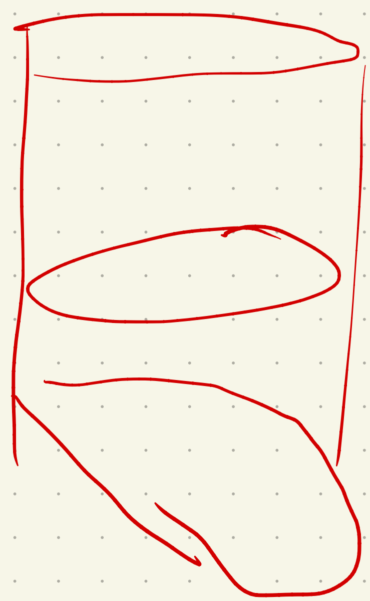
Ω

$z = f_1(x,y)$

$$\iint_R \int_{f_1(x,y)}^{f_2(x,y)} \rho(x,y,z)\,dz\; dA$$

$$=$$

$$\iiint_{\mathcal{E}} \rho(x,y,z)\,dV$$



$\mathcal{E}$ $\quad 2 \le x \le 5/2$

$0 \le y \le \pi$

$0 \le z \le 2$

$$\iiint_{\mathcal{E}} z x \sin(xy)\, dV$$

$$\int_2^{5/2}\int_0^{\pi}\int_0^{2} z x \sin(xy)\, dz\, dy\, dx$$

$$=$$

$$\int_2^{5/2}\int_0^{\pi} x \sin(xy) \int_0^{2} z\, dz\, dy\, dx$$

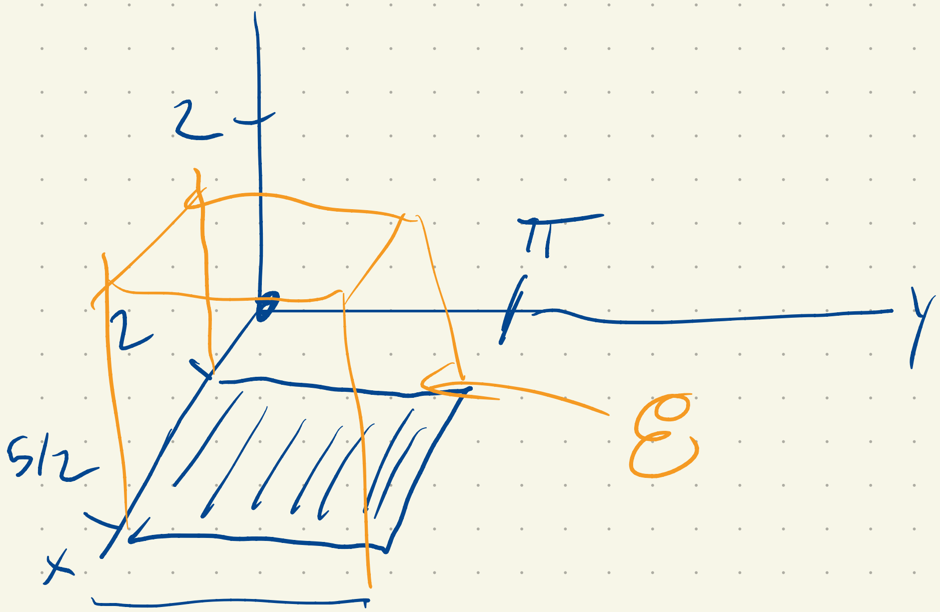
$$=$$

$$\int_2^{5/2}\int_0^{\pi} x \sin(xy)\, \frac{z^2}{2}\Big|_0^2\, dy\, dx$$

$$=$$

$$2\int_2^{5/2}\int_0^{\pi} x \sin(xy)\, dy\, dx$$

$$=$$



$$\int_0^{\pi x} \sin(u)\, du$$

$u = xy$

$y = \pi \Rightarrow u = x\pi$

$y = 0 \Rightarrow u = 0$

$u = xy$

$du = x\,dy$

$$\int x \sin(xy)\,dy = \int \sin(u)\,du$$

$$= -\cos(u)$$

$$= -\cos(xy)$$

$$2 \int_2^{5/2} -\cos(xy) \Big|_{y=0}^{\pi} dx$$

$$\parallel$$

$$2 \int_2^{5/2} -\cos(\pi x) + \cos(0)\,dx$$

$$\parallel$$

$$2 \int_2^{5/2} 1 - \cos(\pi x)\,dx$$

$$\parallel$$

$$2 \left(x - \frac{1}{\pi} \sin(\pi x) \right)_2^{5/2}$$

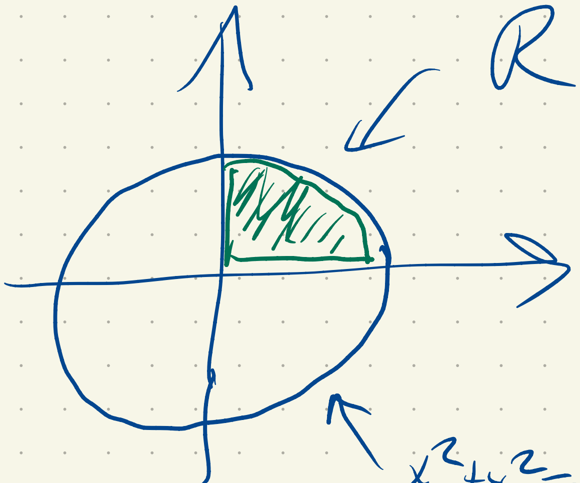
$$\parallel$$

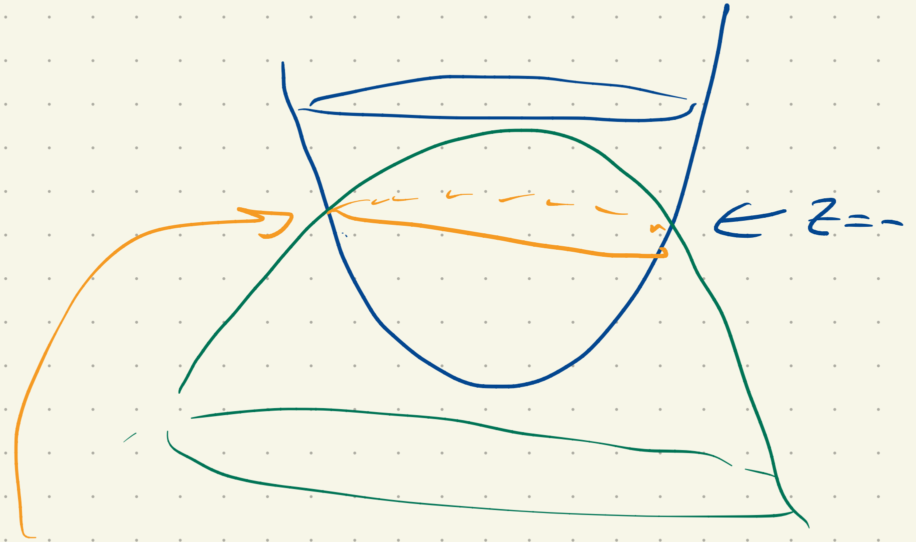
$$1 - \frac{2}{\pi}$$

$\mathcal{E}$ $\quad x \geq 0$

$y \geq 0$

$$3 - x^2 - y^2 \leq z \leq -5 + x^2 + y^2$$





$$3 - x^2 - y^2 = -5 + x^2 + y^2$$

$$2(x^2+y^2) = 8$$

$$x^2+y^2 = 4$$

circle of radius 2

Job: $\displaystyle\iiint_{\mathcal{E}} y \, dV$

$$\iint_R -\int_{3-x^2-y^2}^{-5+x^2+y^2} y \, dz \, dA$$

$$=$$

$$-\iint_R y \; z \Big|_{z=3-x^2-y^2}^{-5+x^2+y^2} dA$$

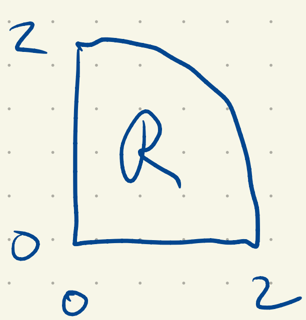
$$=$$

$$-\iint_R y \left(-5+x^2+y^2-(3-x^2-y^2)\right) dA$$

$$=$$

$$-\iint_R \underline{y} \left(-8+2(x^2+y^2)\right) dA$$

$$=$$



$$-\int_0^{\frac{\pi}{2}} \int_0^2 r\sin\theta \left(-8+2r^2\right) \, r \, dr \, d\theta$$

$$\begin{array}{r} 11 \\ +\ 128 \\ \hline 15 \end{array}$$