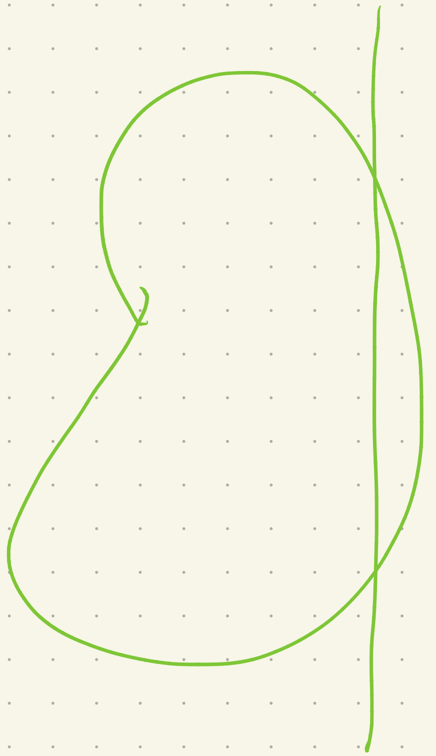


$$g \iint_R x\, \rho(x,y)\, dA$$

$\underbrace{\iint_R x\, \rho(x,y)\, dA}$ $\rightarrow M_y$

first moment of inertia about the y-axis

$$\iint_R (x-a)\, \rho(x,y)\, dA = \iint_R x\, \rho(x,y)\, dA - a \iint_R \rho(x,y)\, dA$$

$$\iint_R x\rho(x,y)\,dA = a\iint \rho(x,y)\,dA$$

$$M_y = a\,m$$

$$a = \frac{M_y}{m} = \bar{x}$$ x component of the centroid of the lamina.

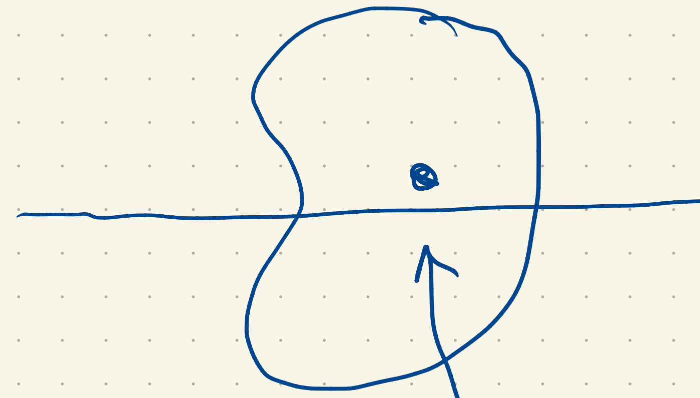
If ρ is constant

$$\bar{x} = \frac{\rho\iint_R x\,dA}{\rho\iint_R dA} = \frac{1}{\text{area}(R)}\iint_R x\,dA$$

$$M_y = \iint_R x\, \rho(x,y)\, dA$$

$$M_x = \iint y\, \rho(x,y)\, dA$$

$$m = \iint_R \rho(x,y)\, dA$$



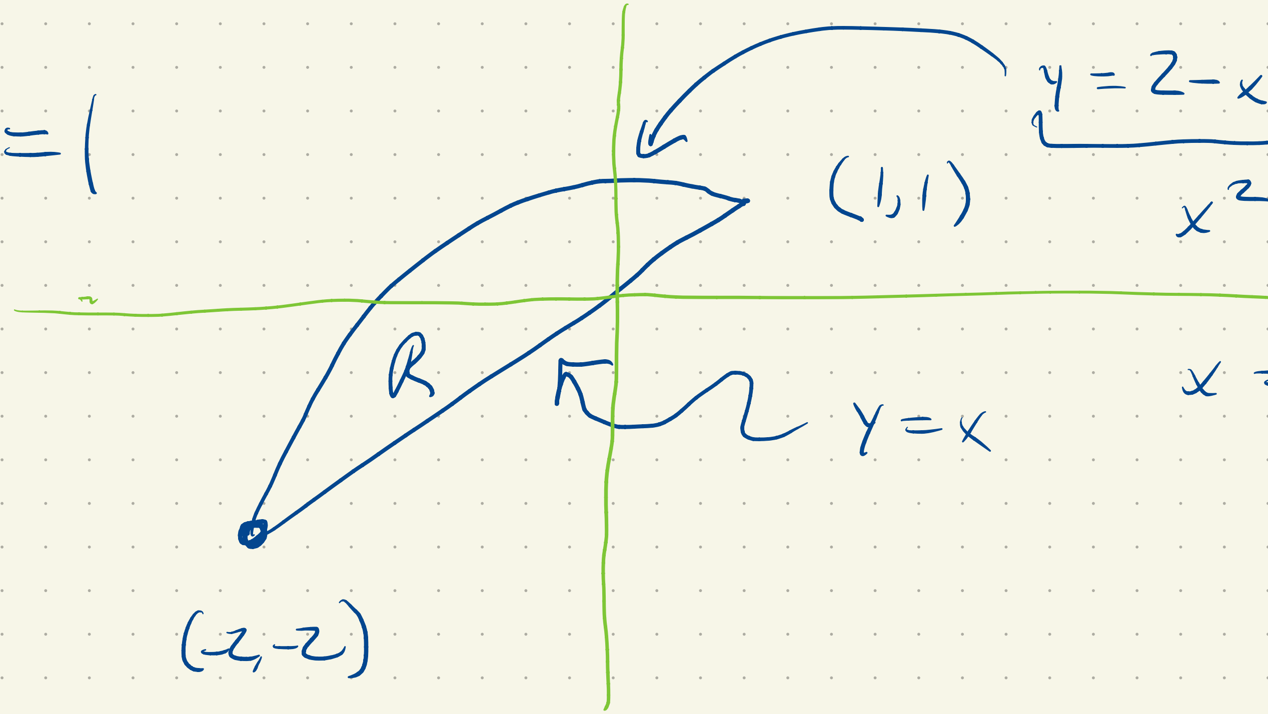
$(\bar{x}, \bar{y})$

centroid

center of mass

$$\bar{x} = \frac{M_y}{m} \qquad \bar{y} = \frac{M_x}{m}$$

E.g. $\rho = 1$



$x^2 = 2 - y$

$x = \sqrt{2-y}$

$\int^{y} \quad dx$

$$M_y = \iint_R x \, dA = \int_{-2}^{1} x \boxed{\int_x^{2-x^2} 1 \, dy} \, dx$$

$$= \int_{-2}^{1} x \left((2 - x^2) - x \right) dx$$

$$= \int_{-2}^{1} 2x - x^3 - x^2 \quad dx$$

$$= -\frac{9}{4}$$

$$m = \iint_R \rho \, dA = \rho \iint_R dA$$

$$= \iint_R dA$$

$$= \int_{-2}^{1} \int_{x}^{2-x^2} 1 \, dy \, dx$$

$$= \int_{-2}^{1} (2-x^2) - x \, dx$$

$$= \frac{9}{2}$$

$$\bar{x} = \frac{M_y}{m} = \frac{-9/4}{9/2} = -\frac{2}{4} = -\frac{1}{2}$$

$$M_x = \iint_R y \, dA = \int_{-2}^{1} \int_{x}^{2-x^2} y \, dy \, dx$$

$$= \int_{-2}^{1} \frac{y^2}{2} \Big|_{y=x}^{2-x^2} dx$$

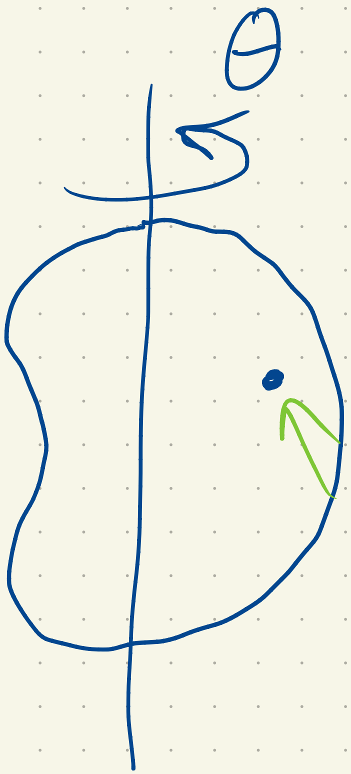
$$= \frac{1}{2} \int_{-2}^{1} (2-x^2)^2 - x^2 \, dx$$

$$= \frac{9}{5}$$

$$\bar{y} = \frac{M_x}{m} = \frac{9/5}{9/2} = \frac{2}{5}$$

Newton's 2nd Law

$$\vec{F} = m \frac{d}{dt} \vec{v}$$



$$[I] = N m s^2$$

$$\frac{d\theta}{dt} = \omega$$

$$\frac{kg\, m}{s^2} \quad m s^2$$

$$kg\, m^2$$

$N m$

$$\tau = I \frac{d}{dt} \omega \qquad \frac{1}{s^2}$$

torque

2nd moment of inertia

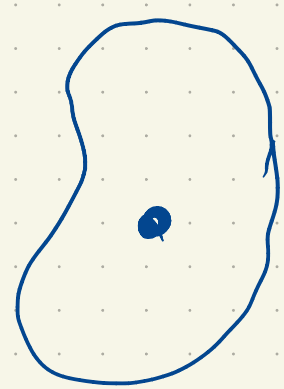
$$I_y = \iint_R x^2 \rho(x,y)\, dA$$ 2nd moment of inertia

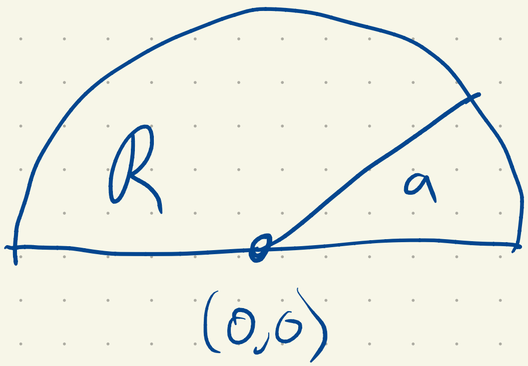
M_y $\qquad$ x

$$I_x = \iint_R y^2 \rho(x,y)\, dA$$

$$I_0 = \iint_R (x^2+y^2) \rho(x,y)\, dA$$

"polar moment"





$\rho = kr$ $\xrightarrow{k}$ constant

job: compute $\bar{y}$

M_x

m

$$M_x = \iint_R y\, kr\, \underbrace{dA}$$

$$= \int_0^{\pi} \int_0^{a} r\sin\theta\, kr\, \underbrace{r}\, dr\, d\theta$$

$$= k\int_0^{\pi} \int_0^{a} r^3 \sin\theta\, dr\, d\theta$$

$$= k \int_0^{\pi} \sin\theta \left.\frac{r^4}{4}\right|_0^a d\theta$$

$$= k \int_0^{\pi} \sin\theta \frac{a^4}{4} d\theta = \frac{k a^4}{4} \int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{k a^4}{2}$$

$$m = \iint_R \rho \, dA = \iint_R k r \, dA$$

$$= \int_0^{\pi} \int_0^{a} k r \; r \, dr \, d\theta$$

$$= \frac{k a^3}{3} \pi$$

$$\bar{y} = \frac{M_x}{m} = \frac{3a}{2\pi}$$