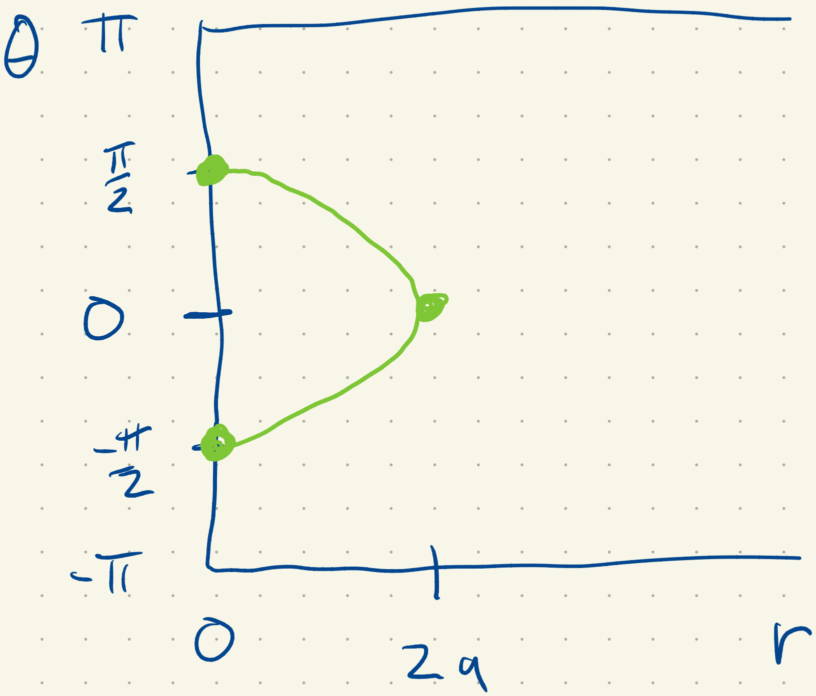
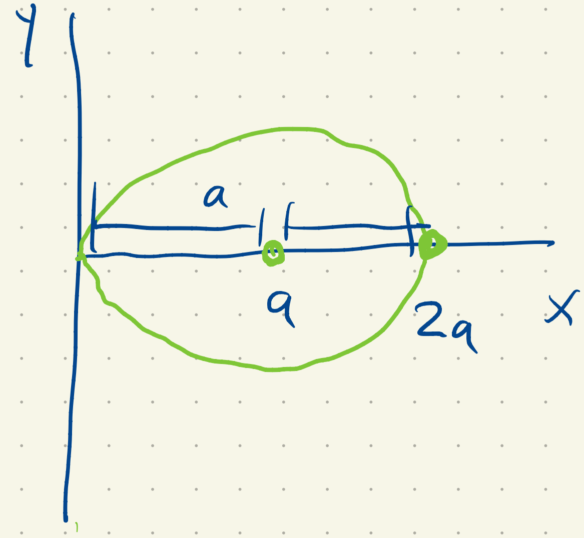
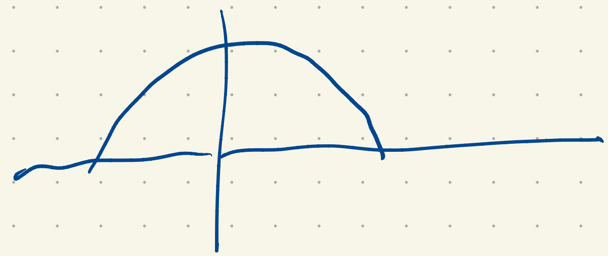




$$r = 2a\cos\theta \qquad a > 0$$

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$



$$r = 2a\cos\theta$$

$$r^2 = 2a \; r\cos\theta$$

$$x^2 + y^2 = 2a\,x$$

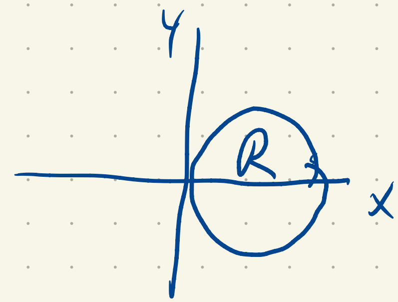
circle centered at $(a,0)$

w/ radius a.

$$x^2 - 2ax + y^2 = 0$$

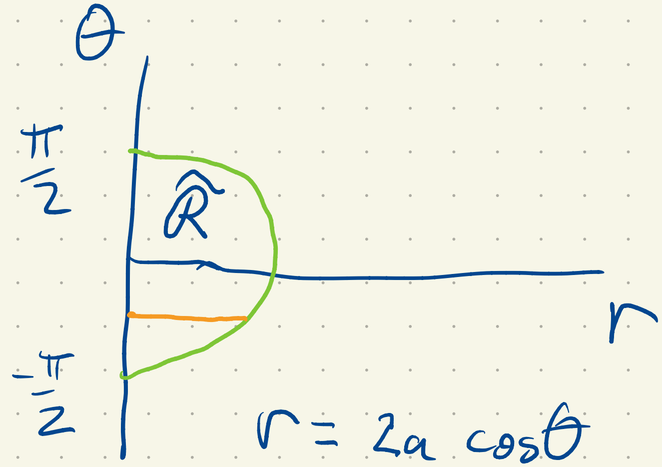
$$x^2 - 2ax + a^2 + y^2 = a^2$$

$$(x-a)^2 + y^2 = a^2$$



Area: $\displaystyle\iint_R 1 \, dA$

$$\boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2a\cos\theta} 1 \quad r \, dr \, d\theta}$$



$r = 2a \cos\theta$

$$\int_{-\frac{\pi}{2}}^{\pi/2} \int_0^{2a\cos\theta} r \, dr \, d\theta = \int_{-\frac{\pi}{2}}^{\pi/2} \left. \frac{r^2}{2} \right|_0^{2a\cos\theta} d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\pi/2} \frac{(2a\cos\theta)^2}{2} \, d\theta$$

$$= 2a^2 \int_{-\pi/2}^{\pi/2} \cos^2\theta \, d\theta$$

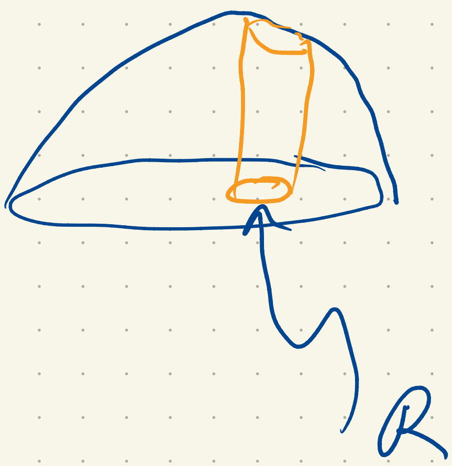
$$= \cancel{2}a^2 \int_{-\pi/2}^{\pi/2} \frac{\cos(2\theta) + 1}{\cancel{2}} \, d\theta$$

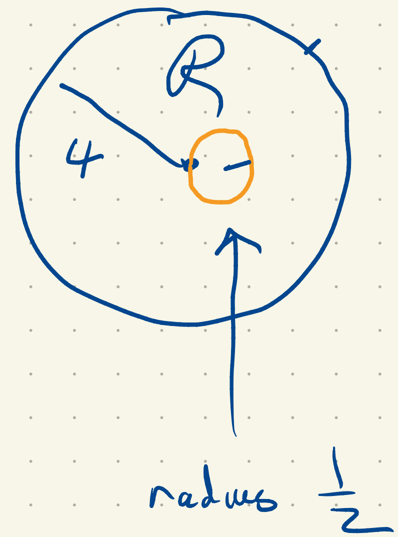
$$= a^2 \left(\frac{\sin(2\theta)}{2} + \theta \right) \Bigg|_{-\pi/2}^{\pi/2}$$

$$= a^2\left(0 + \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)$$

$$= \pi a^2$$

$$16 - x^2 - y^2$$





$$\iint_R (16 - x^2 - y^2)\, dA = V$$

$$r = \cos\theta$$

$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$\int_{-\frac{\pi}{2}}^{\pi/2}\int_{0}^{\cos\theta}\left(16-(r\cos\theta)^2-(r\sin\theta)^2\right)r\,dr\,d\theta$$

$x = r\cos\theta$

$y = r\sin\theta$

$$\int_{-\frac{\pi}{2}}^{\pi/2}\int_{0}^{\cos\theta}\left(16-r^2\left(\cos^2\theta+\sin^2\theta\right)\right)r\,dr\,d\theta$$

$$\int_{-\frac{\pi}{2}}^{\pi/2}\int_{0}^{\cos\theta}\left(16-r^2\right)r\,dr\,d\theta$$

$$\int_{-\frac{\pi}{2}}^{\pi/2}\int_{0}^{\cos\theta}16r-r^3\,dr\,d\theta$$

$$\int_{-\pi/2}^{\pi/2} 8r^2 - \frac{r^4}{4} \Big|_0^{\cos\theta} d\theta$$

$$= \int_{-\pi/2}^{\pi/2} 8\cos^2\theta - \frac{\cos^4\theta}{4} \, d\theta$$

$$\int_{-\frac{\pi}{2}}^{\pi/2} \cos^2\theta \, d\theta = \int_{-\frac{\pi}{2}}^{\pi/2} \frac{\cos(2\theta)+1}{2} \, d\theta = \frac{1}{2}\left[\frac{\sin(2\theta)}{2} + \theta\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] = \frac{\pi}{2}$$

$$\int_{-\pi/2}^{\pi/2} \cos^4\theta \, d\theta = \int_{-\pi/2}^{\pi/2} \left(\frac{1+\cos(2\theta)}{2}\right)^2 d\theta$$

$$= \frac{1}{4}\int_{-\frac{\pi}{2}}^{\pi/2} 1 + 2\cos(2\theta) + \cos^2(2\theta) \, d\theta$$

$$= \frac{1}{4}\left[\pi + \sin(2\theta)\Big|_{-\frac{\pi}{2}}^{\pi/2} + \int_{-\frac{\pi}{2}}^{\pi/2} \cos^2(2\theta)\,d\theta\right.$$

$$= \frac{\pi}{4} + \frac{1}{4}\boxed{\int_{-\pi/2}^{\pi/2} \cos^2(2\theta)\,d\theta}$$

$$\boxed{\int_{-\pi/2}^{\pi/2} \cos^2(2\theta)\,d\theta} = \int_{-\pi/2}^{\pi/2} \frac{\cos(4\theta)+1}{2}\,d\theta = \frac{1}{2}\left[\frac{\sin(4\theta)}{4} + \theta\right]_{-\frac{\pi}{2}}^{\pi/2}$$

$$= \frac{\pi}{2}$$

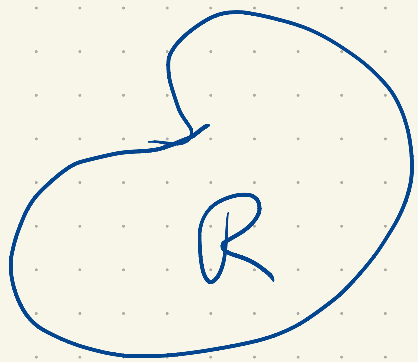
$$\int_{-\pi/2}^{\pi/2} \cos^4\theta\,d\theta = \frac{\pi}{4} + \frac{1}{4}\,\frac{\pi}{2} = \frac{3\pi}{8}$$

$$\int_{-\pi/2}^{\pi/2} 8\cos^2\theta - \frac{\cos^4\theta}{4}\,d\theta = 8 \cdot \frac{\pi}{2} - \frac{1}{4}\,\frac{3\pi}{8} = 4\pi - \frac{3\pi}{32}$$

$$= \boxed{\frac{125\pi}{32}}$$

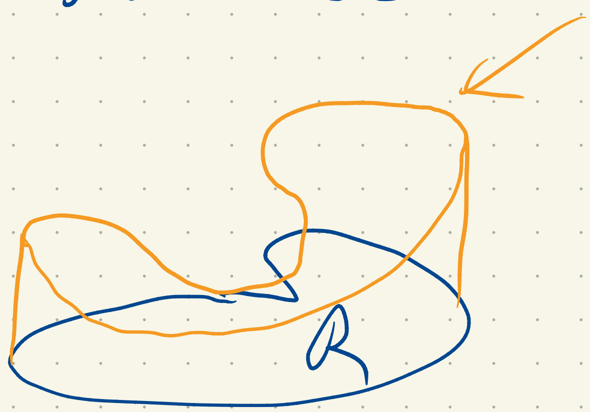
5.6 (Applications)

1) 2-d areas



$$\text{area} = \iint_R 1 \, dA$$

2) 3-d volumes



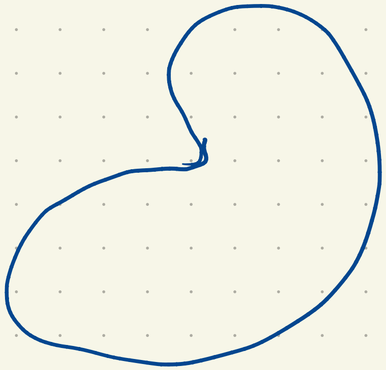
$z = f(x,y)$

$$\text{volume} = \iint_R f(x,y) \, dA$$

3) Average values

temperature

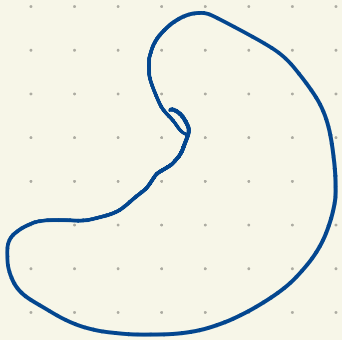
$T(x,y)$



$$\frac{1}{\text{area}(R)} \iint_R T(x,y)\, dA$$

4) density $\rightarrow$ mass

$\rho(x,y)$ (mass/area)

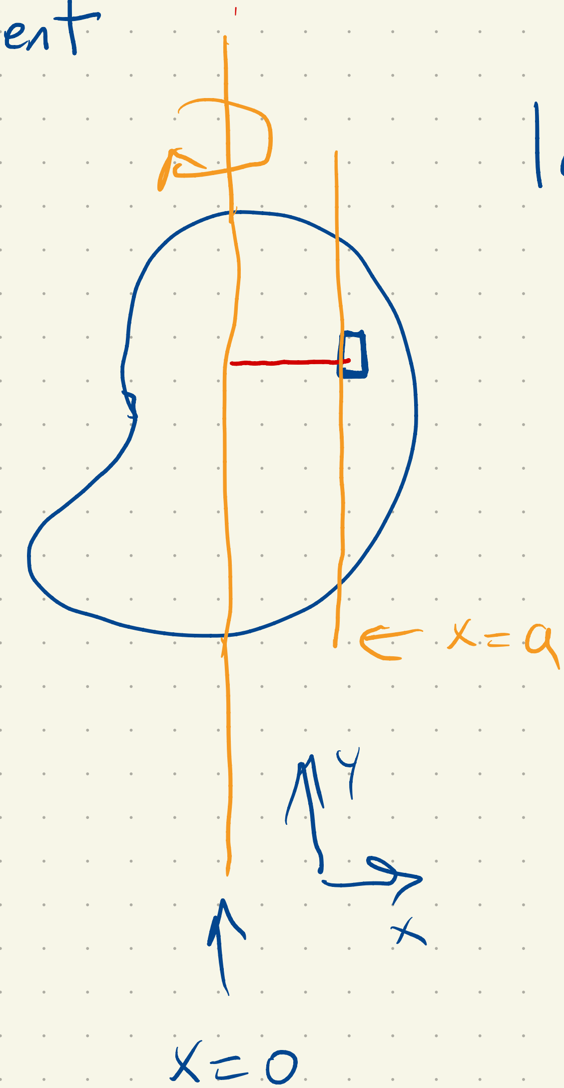


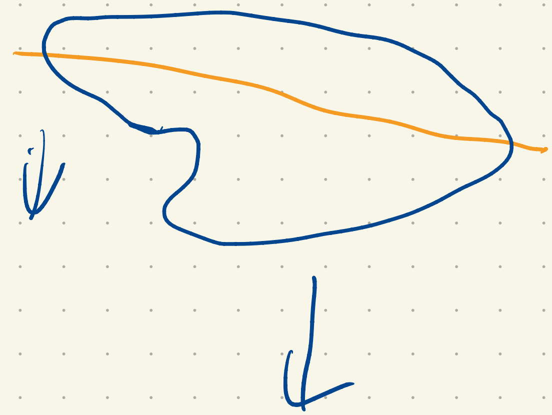
$$\text{mass} = \iint_R \rho(x,y)\, dA$$

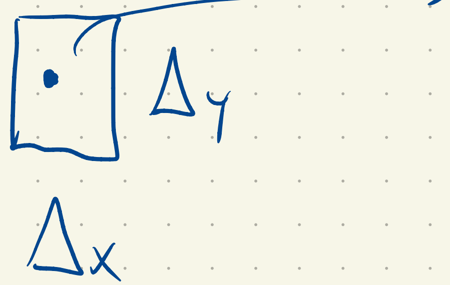
Moment

density $\rho(x,y)$

lamina







(x_{ij}, y_{ij})

$\rho(x_{ij}, y_{ij})$

mass $\approx \rho(x_{ij}, y_{ij})\,\Delta x\,\Delta y$

Force due to gravity mass $\cdot\, g \rightarrow$ grav. acceleration force on squne $\quad g\,\rho(x_{ij}, y_{ij})\,\Delta x\,\Delta y$

τ_0 Torque due to the piece

$$x_{ij}\, g\, \rho(x_{ij}, y_{ij})\,\Delta x\,\Delta y$$

Total terque

$$\sum_{i,j} x_{ij}\, g\, \rho(x_{ij}, y_{ij})\,\Delta x\,\Delta y$$

Total torque $\quad \iint_R x\, g\, \rho(x,y)\, dA$

$$\underbrace{g \iint_R x\, \rho(x,y)\, dA}_{} \to M_y$$

first moment of inertia about the y-axis

$$\iint_R (x-a)\,\rho(x,y)\,dA = \iint_R x\,\rho(x,y)\,dA - a\iint_R \rho(x,y)\,dA$$