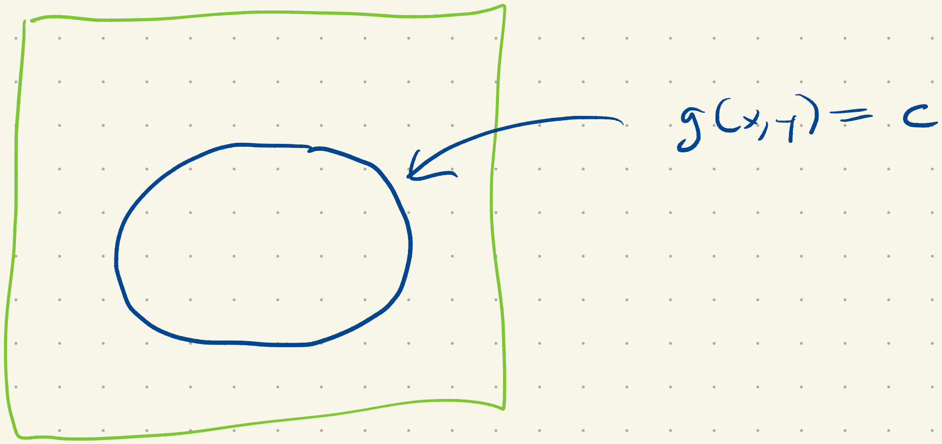


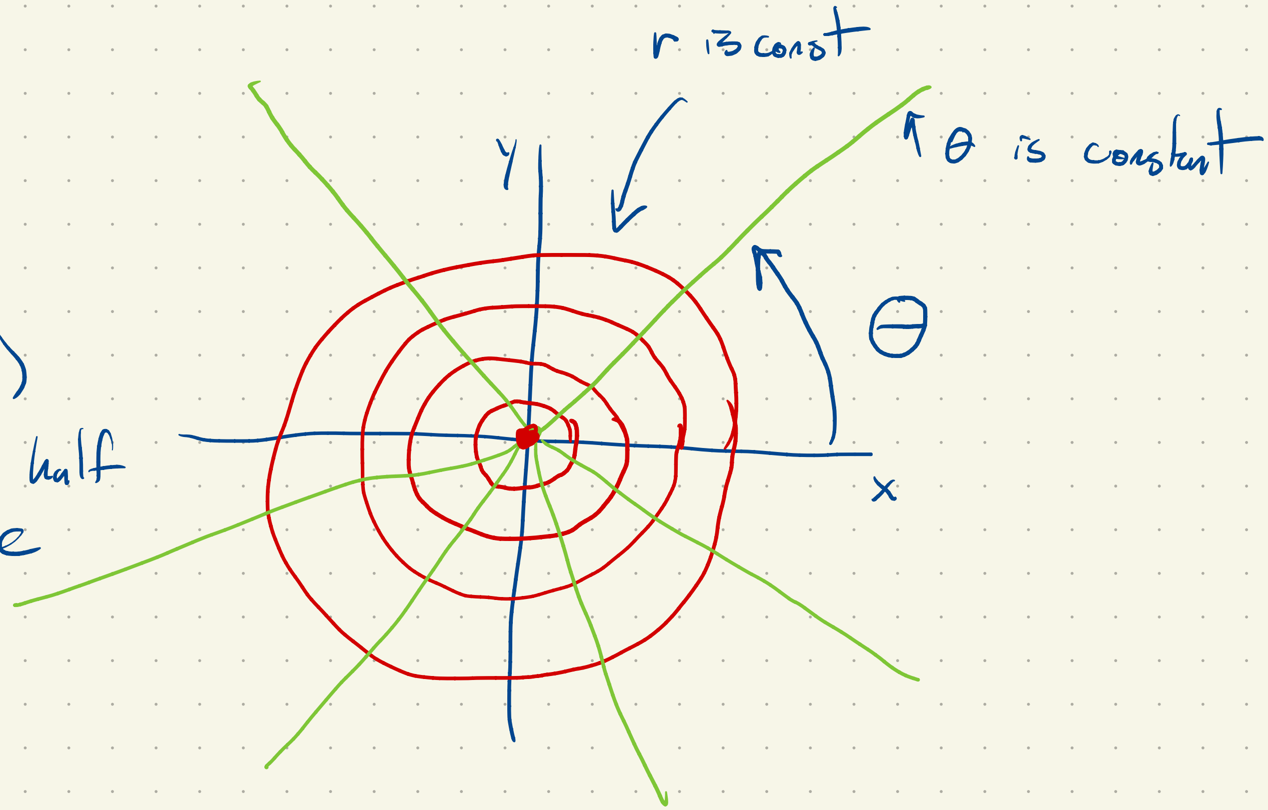
Polar Coordinates

(x,y) $\quad$ (r,θ)

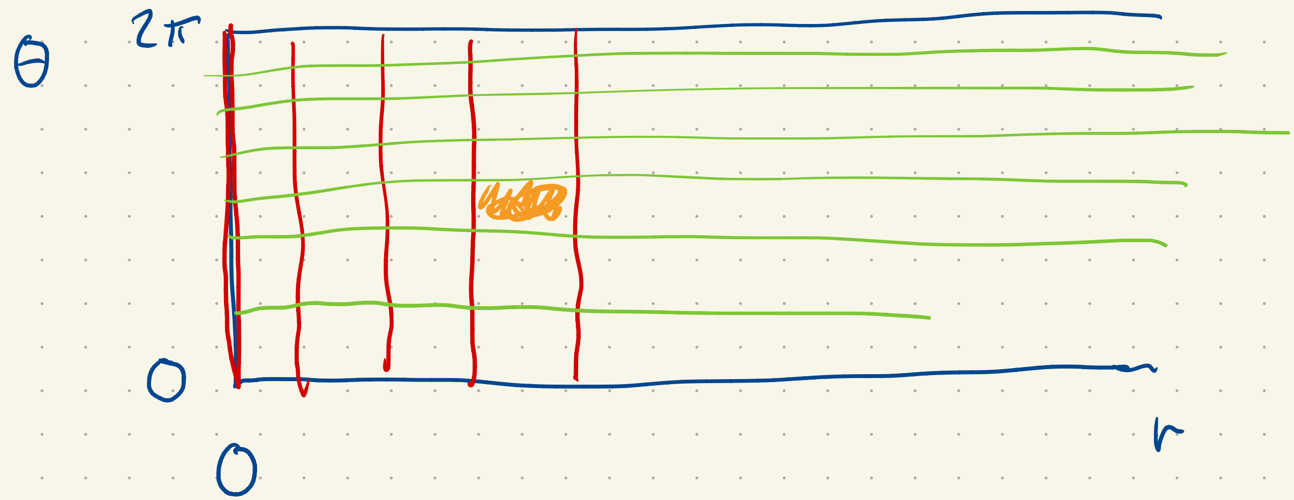
$x = r\cos\theta, \quad y = r\sin\theta$

$r = \sqrt{x^2+y^2} \quad \theta = \mathrm{atan2}(y,x)$

$= \arctan(y/x)$ half the time

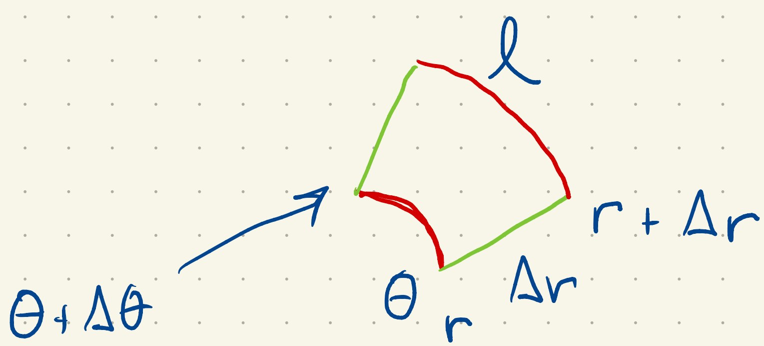


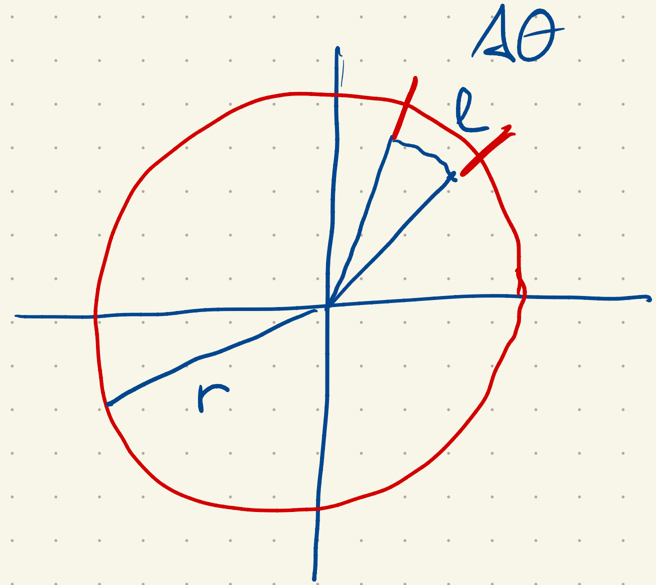
(r, θ)



$\Delta\theta = \frac{\pi}{2}$

$= \frac{2\pi}{4}$





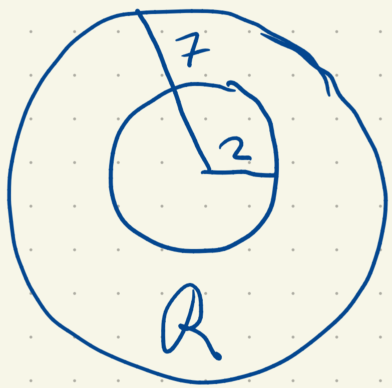
$\ell \Delta r$

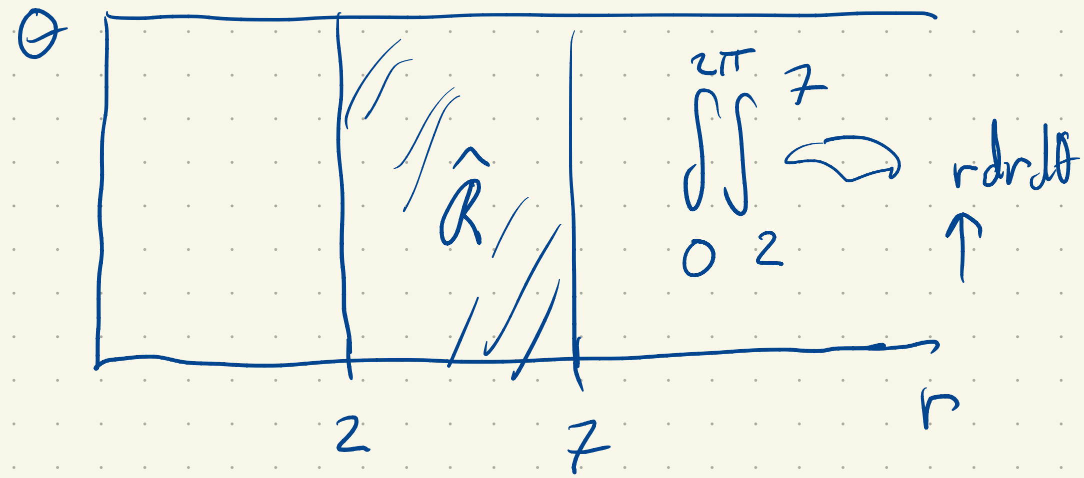
all the way around length $2\pi r$

Area is approx $\ell \Delta r = r \Delta\theta \Delta r$

$\ell = \Delta\theta\, r$

$$l = \Delta\theta\, r = \frac{\Delta\theta}{2\pi}\, 2\pi r$$





$$\iint_R f(x,y)\, \underbrace{dA(x,y)}_{dx\,dy}$$

$$\iint_{\hat{R}} f(r\cos\theta, r\sin\theta)\, \underbrace{d\hat{A}}_{r\,dr\,d\theta}$$

$u = x^2$

$du = 2x\,dx$

$$\iint_R 1\, dA \quad (\text{area of } R)$$

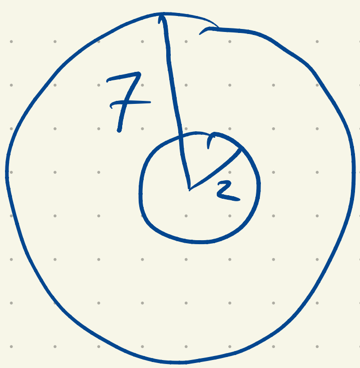
$$\int_0^{2\pi}\int_2^7 1 \quad r\,dr\,d\theta$$

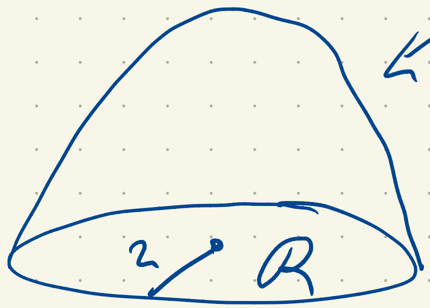
$$\int_0^{2\pi}\int_2^7 r\,dr\,d\theta = \int_0^{2\pi} \frac{r^2}{2}\Big|_2^7 d\theta$$

$$= \int_0^{2\pi} \frac{7^2}{2} - \frac{2^2}{2}\, d\theta$$

$$= 2\pi\left(\frac{7^2}{2} - \frac{2^2}{2}\right)$$

$$= \pi 7^2 - \pi 2^2$$





$4 - x^2 - y^2$

$V = 8\pi$

$$\iint_R 4 - x^2 - y^2 \, dA$$

$-(x^2+y^2)$

$$\int_0^{2\pi}\int_0^{2} (4 - r^2)\, r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{2} 4r - r^3\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{4r^2}{2} - \frac{r^4}{4}\Big|_0^2 \,d\theta$$

$$= \int_0^{2\pi} 2\cdot 2^2 - \frac{2^4}{4}\,d\theta$$

$$= \int_0^{2\pi} 8 - 4 \, d\theta$$

$$= 4 \int_0^{2\pi} d\theta$$

$$= 4 \cdot 2\pi$$

$$= 8\pi$$