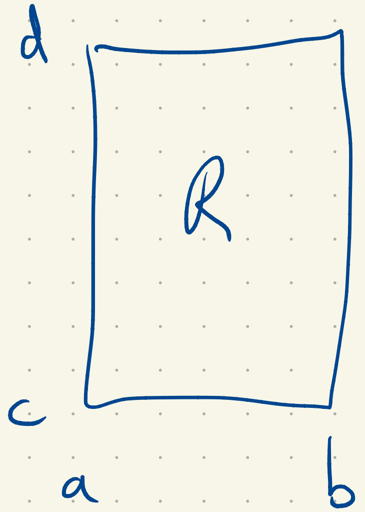
Last class: double integrals over rectangles



$f(x,y)$

$$\iint_R f(x,y)\, dA(x,y)$$

$$= \int_c^d \int_a^b f(x,y)\, dx\, dy$$

$$= \int_a^b \int_c^d f(x,y)\, dy\, dx$$

$R = [1,2] \times [0,\pi]$

$$\iint_R y \sin(xy)\, dA$$

$$\int_0^{\pi} \int_1^2 y \sin(xy)\, dx\, dy \qquad A$$

$$\int_1^2 \int_0^{\pi} y \sin(xy)\, dy\, dx \qquad B$$

$$\int_0^{\pi} y \sin(7y)\,dy$$

$$\int_0^{\pi} \int_1^{2} y \sin(xy)\,dx\,dy$$

$u = yx$

$du = y\,dx$

$$\int_0^{\pi} \int_y^{2y} \sin(u)\,du\,dy$$

$$\int_0^{\pi} -\cos(u) \Big|_y^{2y} dy = \int_0^{\pi} -\cos(2y) + \cos(y)\,dy$$

$$= -\frac{1}{2}\sin(2y) + \sin(y) \Big|_0^{\pi}$$

$$= 0$$

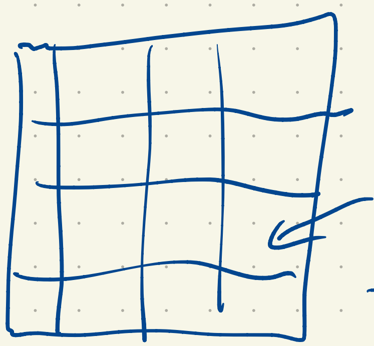
Applications   integrate density to get mass

integrate height to get volume.

$$f(x,y) = 2 - x^2 - y^2$$

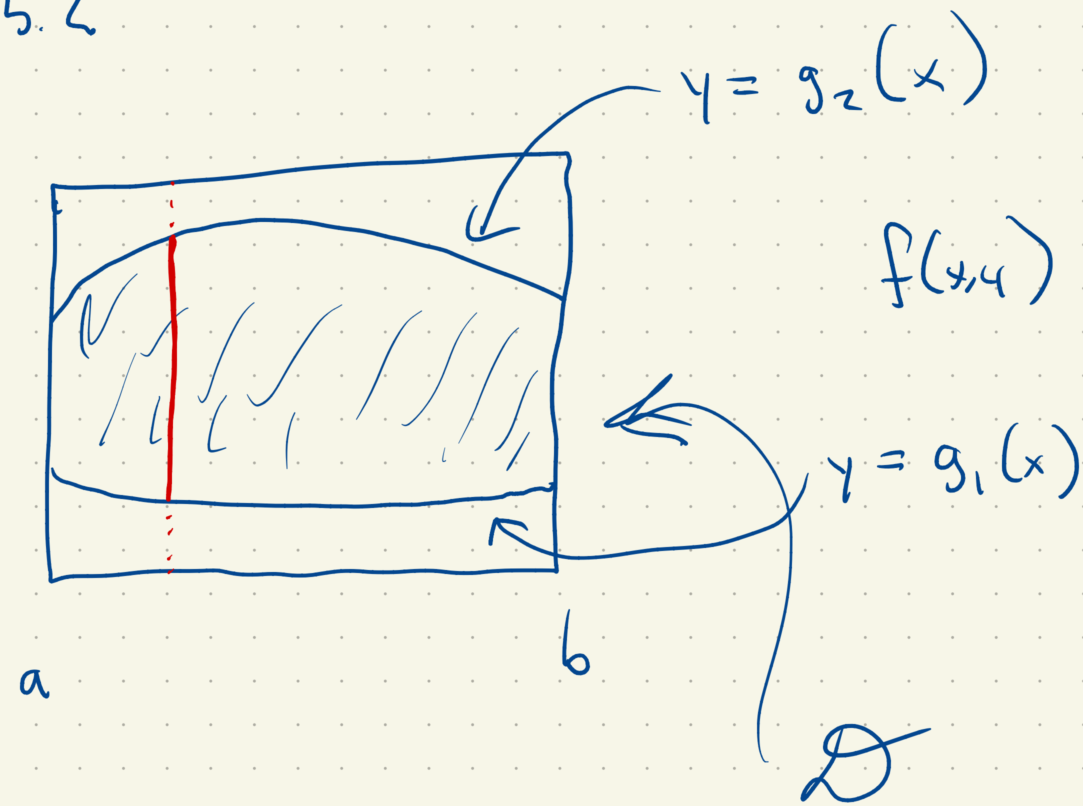
$$\iint_R 2 - x^2 - y^2 \, dA(x,y)$$

is volume of the region between $z=0$ and $z=f(x,y)$ over R.

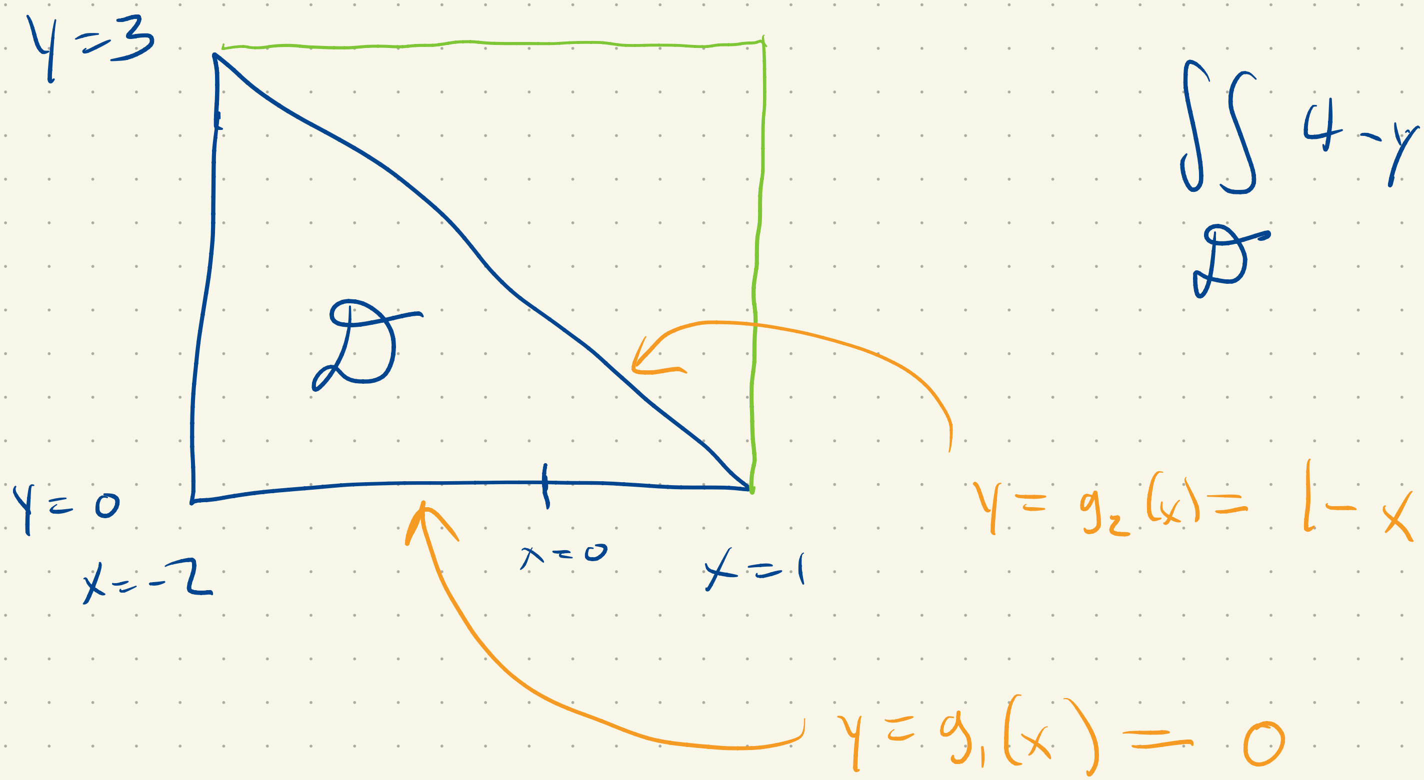


$f(x_i, y_j)\Delta x \Delta y$

Sec. 5.2



$$\iint_D f(x,y)\, dA(x,y) = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx$$

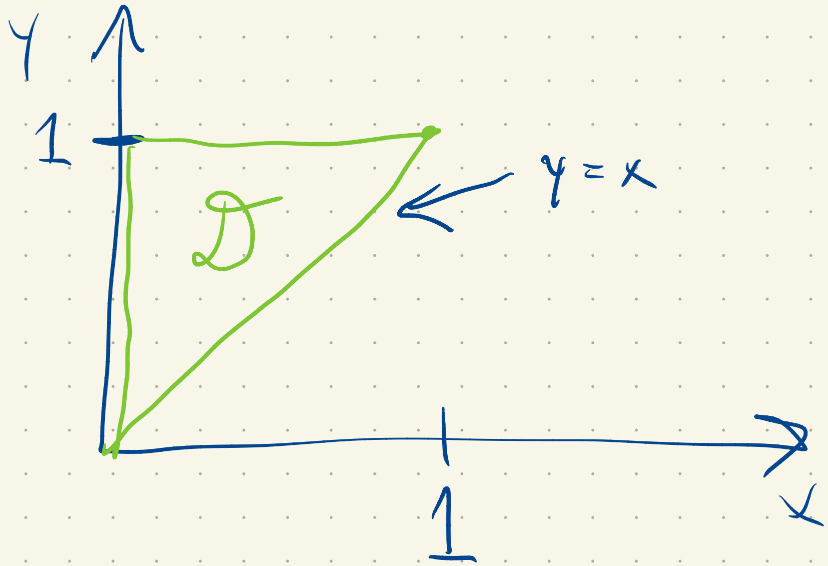


$$\iint_D 4-y \, dA(x,y)$$

$$\int_{-2}^{1}\int_{0}^{1-x} 4-y \, dy \, dx = \int_{-2}^{1} 4y - \frac{y^2}{2} \Big|_{y=0}^{1-x} dx$$

$$= \int_{-2}^{1} 4(1-x) - \frac{(1-x)^2}{2} \, dx$$

$$= \frac{27}{2}$$



$f(x,y) = \underline{\sin(y^2)}$

$$\int_0^7 \underline{\sin(y^2)}\, dy$$

$$\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx$$

$$\int_0^y \sin(t^2)\, dt$$

$$\int_0^1 \int_0^y \sin(y^2)\, dx\, dy = \int_0^1 \sin(y^2)\, x \Big|_{x=0}^{y} dy$$

$$= \int_0^1 y \sin(y^2)\, dy$$

$u = y^2$

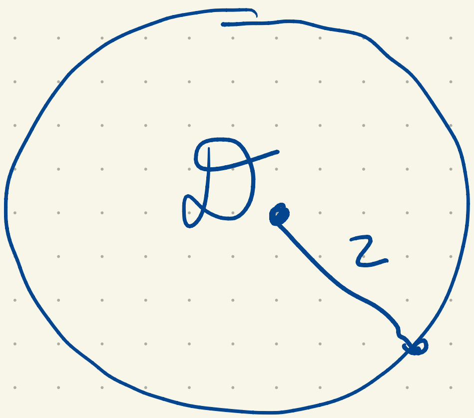
$du = 2y\, dy$

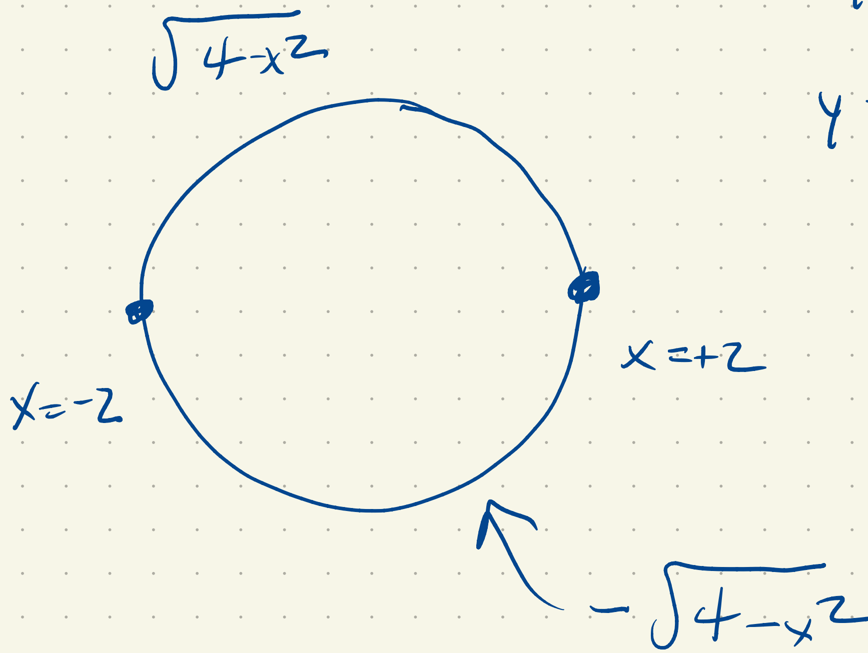
$$= \frac{1}{2}\left[1 - \cos(1)\right]$$

$$f(x,y) = 4 - x^2 - y^2$$

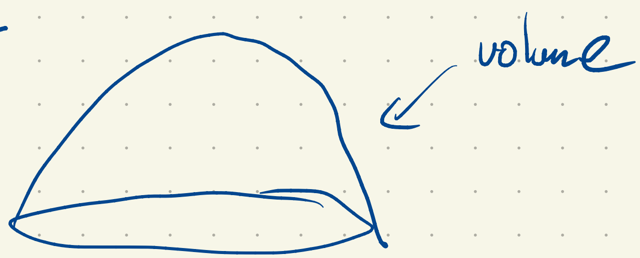
$$x^2 + y^2 = 4$$

$$y = \pm\sqrt{4-x^2}$$





$$\iint_D 4 - x^2 - y^2 \, dA$$



$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 4 - x^2 - y^2 \, dy \, dx$$

$$= \int_{-2}^{2} (4-x^2)y - \frac{y^3}{3} \Bigg|_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx$$

$$= 2\int_{-2}^{2} (4-x^2)\sqrt{4-x^2} - \frac{1}{3}\left(\sqrt{4-x^2}\right)^3 dx$$

$$= \frac{4}{3}\int_{-2}^{2} (4-x^2)^{3/2} dx$$

$$x = 2\sin\theta$$

$$dx = 2\cos\theta\, d\theta$$

$$= \frac{4}{3}\int_{-\frac{\pi}{2}}^{\pi/2} (4\cos^2\theta)^{3/2} 2\cos\theta\, d\theta$$

$$4-x^2 = 4-4\sin^2\theta$$

$$= 4(1-\sin^2\theta)$$

$$= 4\cos^2\theta$$

$$= \frac{4}{3}\int_{-\frac{\pi}{2}}^{\pi/2} 8\cos^3\theta\cos\theta\, d\theta$$

$$= \frac{64}{3}\int_{-\pi/2}^{\pi/2} \cos^4\theta\, d\theta$$

$$= \frac{64}{3}\int_{-\pi/2}^{\pi/2} \frac{3}{8} + \frac{1}{2}\cos(2\theta) + \cos(4\theta)\, d\theta$$

$$= \frac{64}{3}\left[\frac{3}{8}\theta + \frac{1}{4}\sin(2\theta) + \frac{1}{32}\sin(4\theta)\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{64}{3}\left[\frac{3}{8}\frac{\pi}{2} - \frac{3}{8}\left(-\frac{\pi}{2}\right)\right]$$

$$= 8\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = \underline{\underline{8\pi}}$$