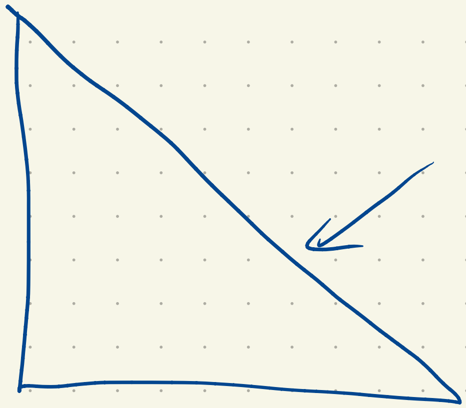


$x + y = 9$

$y = 9 - x$

$f(x, y) = x^2 - y^2$

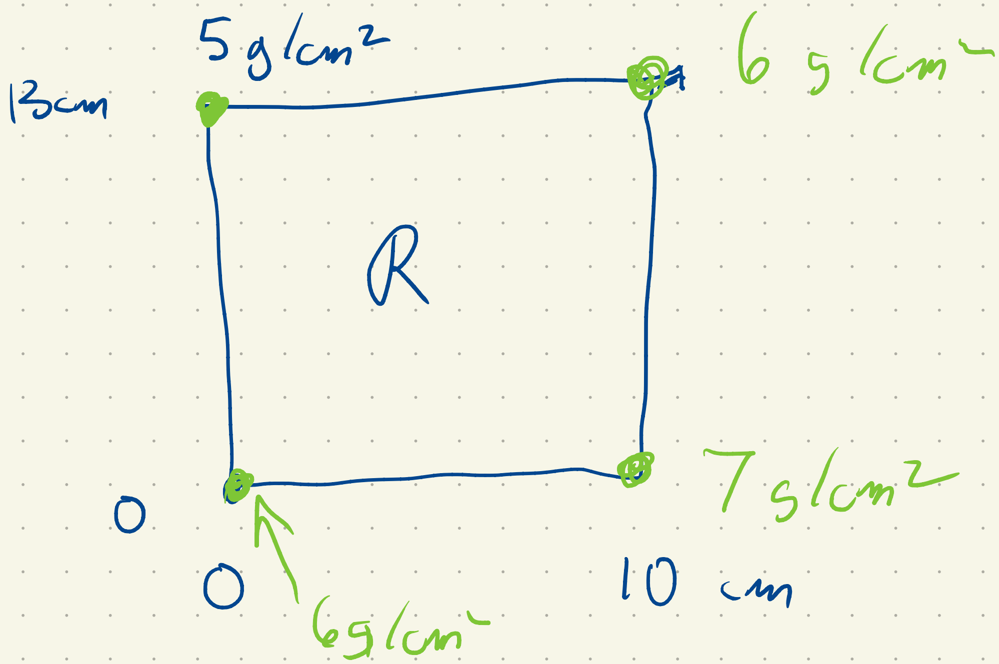
$x^2 - (9-x)^2$

Integration

$\boxed{1\ g/mL}$

$1\ g/cm^3$

$\dfrac{\text{mass}}{\text{volume}}$



planar density g/cm^2

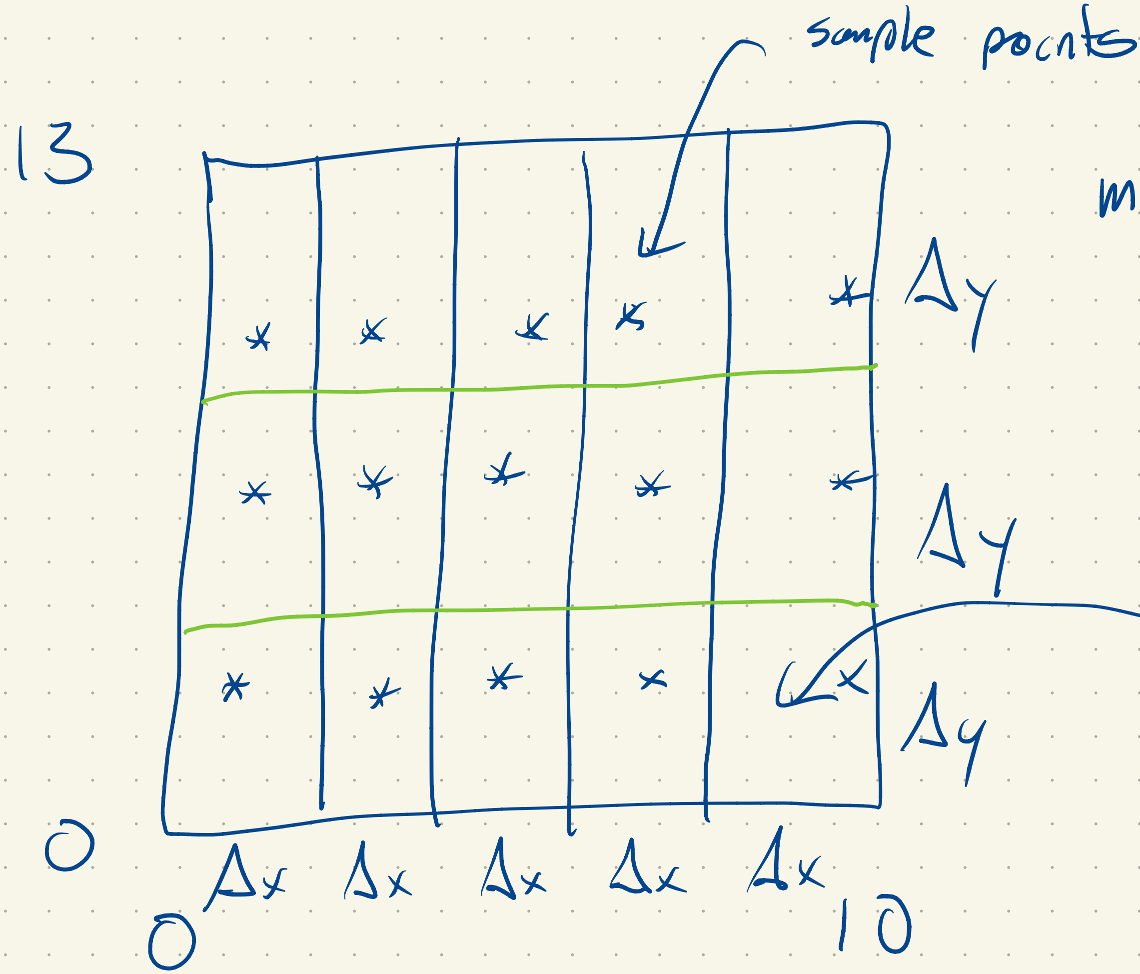
$\rho = 5\ g/cm^2$

area: $13 \cdot 10 = 130\ cm^2$

mass $= 650\ g$

$$\rho(x,y) = \left(6 + \frac{x}{10} - \frac{y}{13}\right) g/cm^2$$

Task: determine the mass



m pieces of width Δy

$$\Delta y = \frac{13}{m}$$

area $\Delta x \Delta y$

$i = 3, j = 2$

n pieces of width Δx

$$\Delta x = \frac{10}{n}$$

x_{ij}, y_{ij} $\quad$ i^{th} j^{th} box

$1 \leq i \leq n$

$1 \leq j \leq m$

$x_i \quad 1 \leq i \leq n$

$y_j \quad 1 \leq j \leq m$

(x_i, y_j) in i^{th}, j^{th} rectangle

(i, j)

Approximate mass $\displaystyle\sum_{j=1}^{m}\sum_{i=1}^{n} \rho(x_i, y_j)\,\Delta x\,\Delta y$

(x_i, y_j)

Δ

Do a better job by taking m, n really large.

$m, n \to \infty$?

area

$$\iint_R \rho(x,y)\, dA(x,y)$$

$$\int_0^{13}\int_0^{10} 6 + \frac{x}{10} - \frac{y}{13}\, dx\, dy$$

$$\sum_{i=1}^{n} \rho(x_i, y_i)\, \Delta x$$

Fubini's Theorem

If $f(x,y)$ is continuous on $R = [a,b] \times [c,d]$

then
$$\iint_R f(x,y)\, dA(x,y) = \int_c^d \left[\int_a^b f(x,y)\, dx\right] dy$$
$$= \int_a^b \left[\int_c^d f(x,y)\, dy\right] dx$$

iterated integrals

$$\text{"}\lim_{n\to\infty}\text{"} \sum_{i=1}^{n} \rho(x_i, y_j)\, \Delta x = \int_0^{10} \rho(x, y_j)\, dx$$

$$\iint$$

$$\text{mass} = \iint_R \rho(x,y)\, dA(x,y)$$

$$= \int_0^{13} \boxed{\int_0^{10} 6 + \frac{x}{10} - \frac{y}{13}\, dx}\, dy$$

$$= \int_0^{13} 6x + \frac{x^2}{20} - \frac{xy}{13} \Big|_{x=0}^{10}\, dy$$

$$= \int_0^{13} 60 + \frac{100}{20} - \frac{10y}{13} \, dy$$

$$= \int_0^{13} 65 - \frac{10}{13} y \, dy$$

$$= 65y - \frac{10}{2 \cdot 13} y^2 \Big|_0^{13}$$

$$= 65 \cdot 13 - \frac{5}{13} \cdot 13^2$$

$$= 65 \cdot 13 - 5 \cdot 13$$

$$= 60 \cdot 13 = 780 \text{ g}$$

Average density $\frac{1}{\text{area}} \iint_R \rho(x,y)\, dA(x,y)$

$$\frac{780}{130} \frac{g}{cm^2} = \frac{78}{13} = 6\ g/cm^2$$