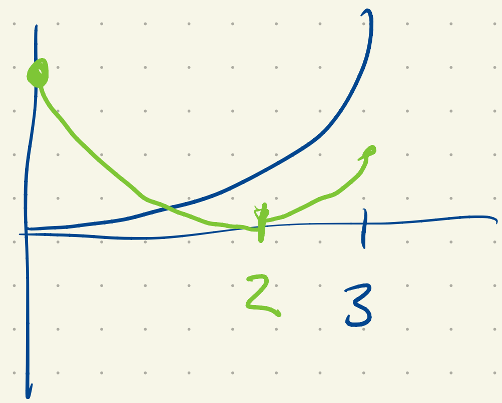
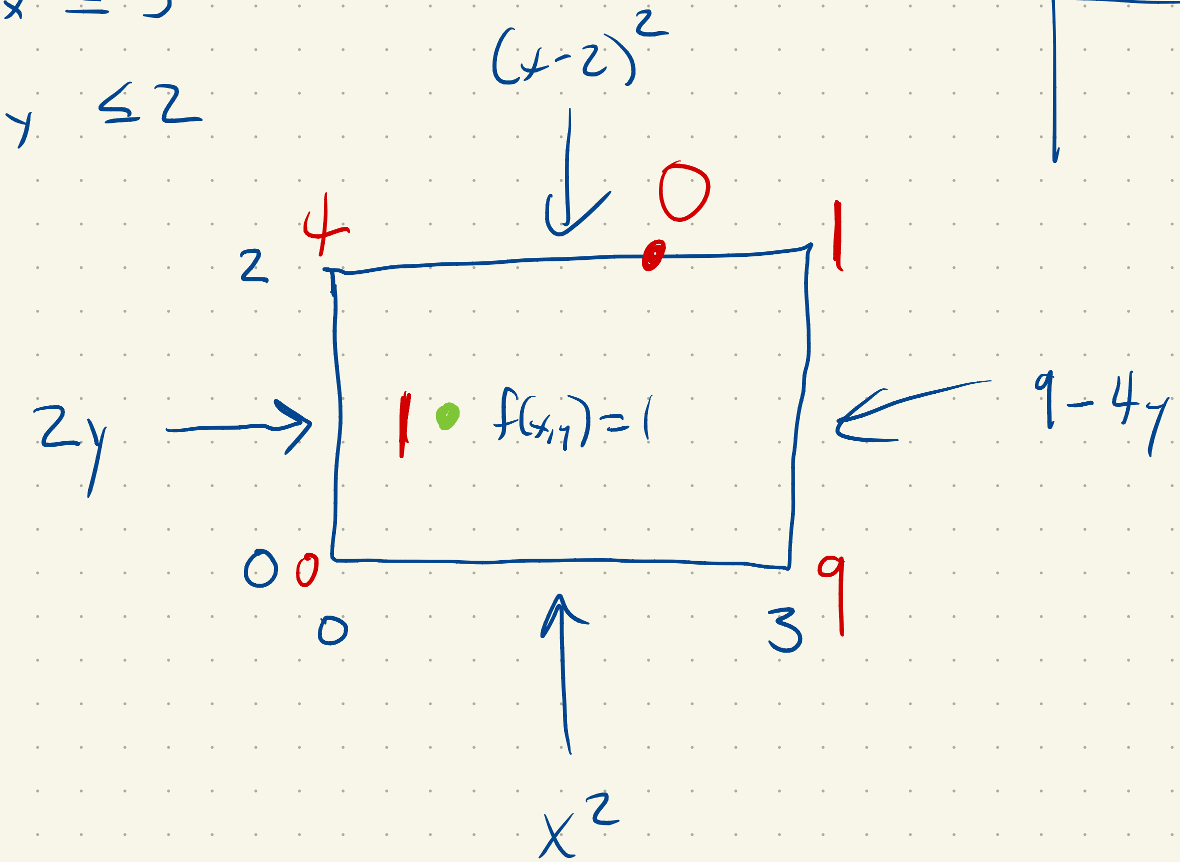
$$f(x,y) = x^2 - 2xy + 2y$$

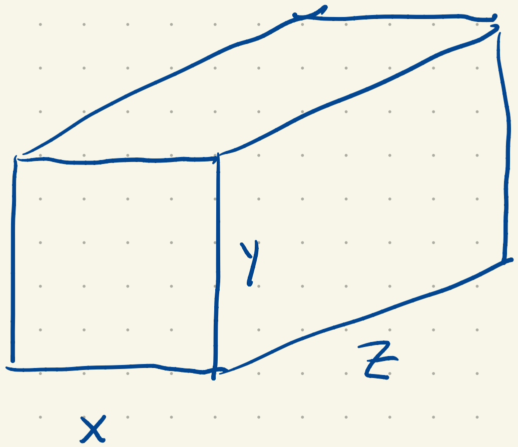


$$0 \le x \le 3$$

$$0 \le y \le 2$$



Lagrange Multipliers



Volume $V = xyz$

girth $x + y + x + y = 2x + 2y$

length z

Restriction girth + length ≤ 108 in

$$2x + 2y + z \leq 108$$

Maximize volume of a shippable box.

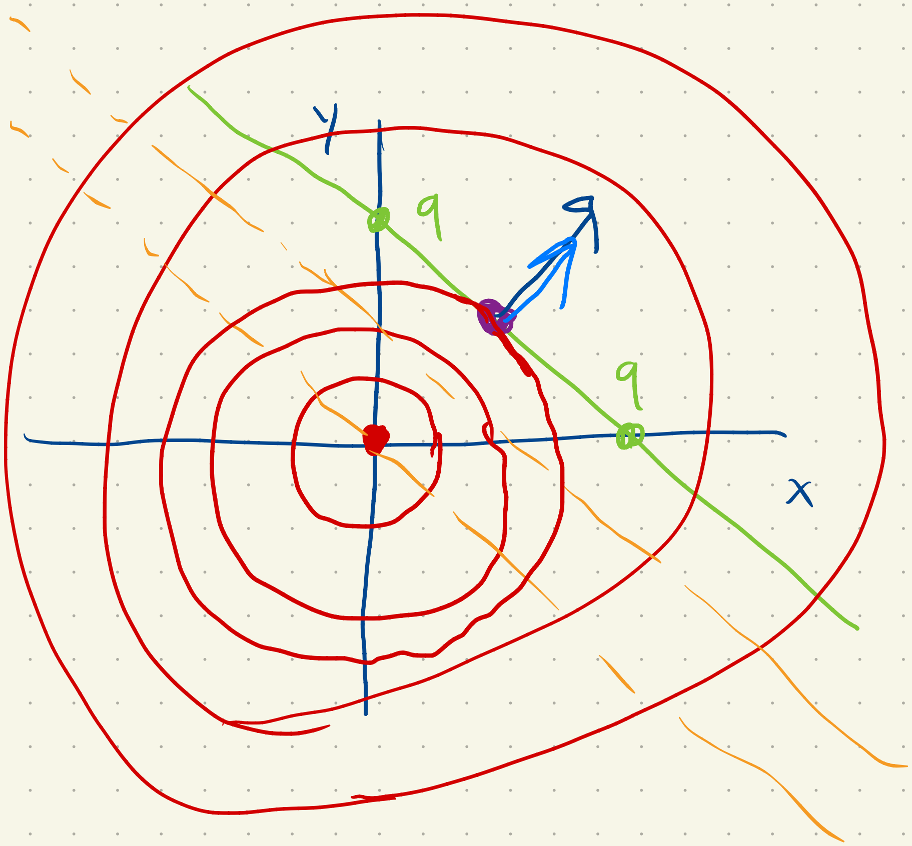
$$2x + 2y + z = 108 \quad \text{] constraint}$$

Maximize $V = xyz$ subject to this constraint

Easier example:

Minimize $f(x,y) = x^2 + y^2$ subject to

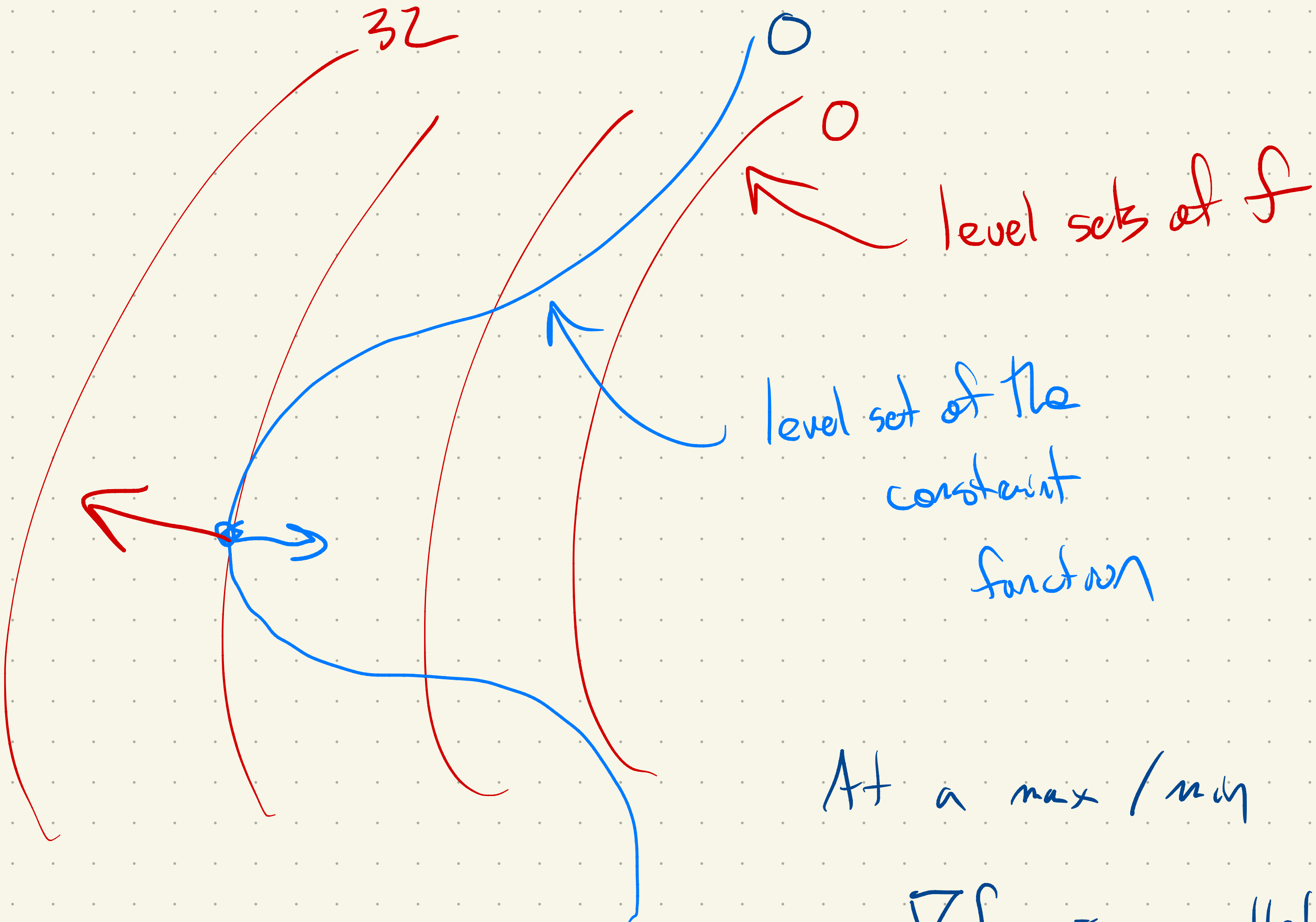
$\underbrace{x + y}_{g(x,y)} = 9$ constraint function



$g(x,y) = 0$

$x + y = 0$

$y = -x$



At a max / min

∇f is parallel to ∇g

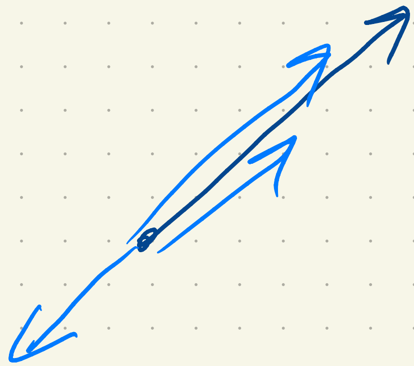
$f(x,y) = x^2 + y^2$

$g(x,y) = x + y$

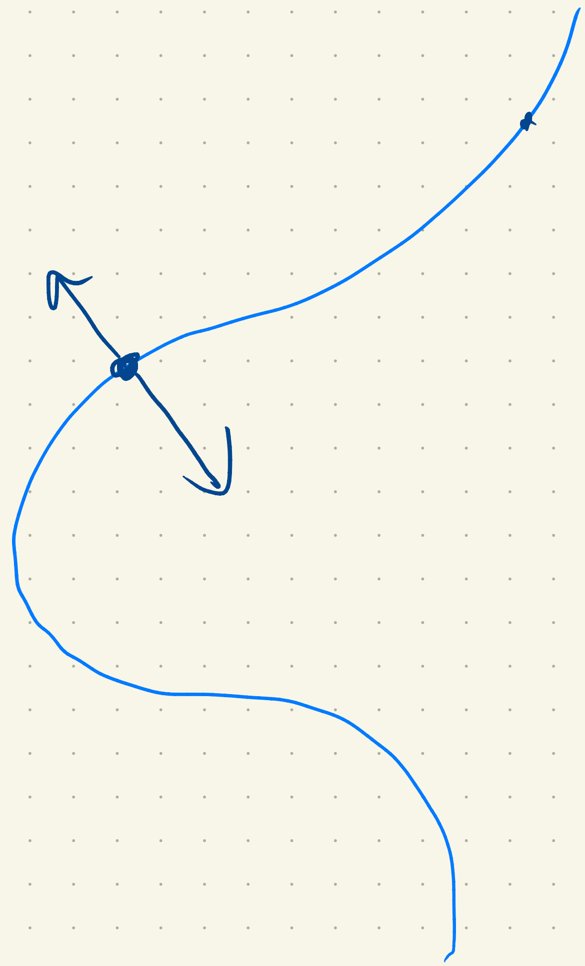
Looking for (x_0, y_0) where

1) ∇f is parallel to ∇g at (x_0, y_0)

2) $g(x_0, y_0) = 9$



$$\vec{\nabla} f(x_0,y_0) = \underbrace{\lambda}_{\text{Lagrange Multiplier}} \vec{\nabla} g(x_0,y_0)$$



$$\frac{\partial f}{\partial x}(x_0,y_0) = \lambda \frac{\partial g}{\partial x}(x_0,y_0)$$

$$\frac{\partial f}{\partial y}(x_0,y_0) = \lambda \frac{\partial g}{\partial y}(x_0,y_0)$$

$$g(x_0,y_0) = 9$$

$$f(x,y) = x^2 + y^2$$

$$g(x,y) = x + y$$

$$\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial f}{\partial y} = 2y$$

$$\frac{\partial g}{\partial x} = 1 \qquad \frac{\partial g}{\partial y} = 1$$

$$\frac{\partial f}{\partial x}(x_0,y_0) = \lambda \frac{\partial g}{\partial x}(x_0,y_0)$$

$$\frac{\partial f}{\partial y}(x_0,y_0) = \lambda \frac{\partial g}{\partial y}(x_0,y_0)$$

$$g(x_0,y_0) = 9$$

$$2x = \lambda \cdot 1$$

$$2y = \lambda 1$$

$$x + y = 9$$

$$2x = \lambda \Rightarrow x = \lambda/2$$

$$2y = \lambda \Rightarrow y = \lambda/2$$

$$x + y = 9 \Rightarrow \frac{\lambda}{2} + \frac{\lambda}{2} = 9 = \lambda = 9$$

$$\Rightarrow x = 9/2$$

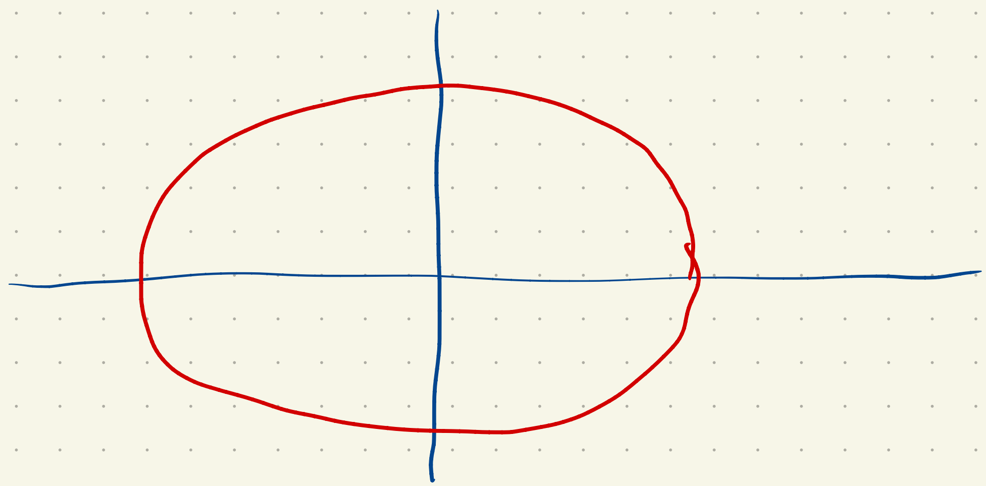
$$y = 9/2$$

$$f\left(\tfrac{9}{2}, \tfrac{9}{2}\right) = \frac{9^2}{4} + \frac{9^2}{4} = 2 \cdot \frac{81}{4} = \frac{81}{2}$$

min value.

$$f(x,y) = x^2 + 4y^3$$

Find min/max of f on the ellipse $x^2 + 2y^2 = 1$



$$g(x,y) = x^2 + 2y^2$$

$$x + y = 9$$

$$g(x,y) = x + y$$

$$\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial f}{\partial y} = 12y^2 \qquad \frac{\partial g}{\partial x} = 2x \qquad \frac{\partial g}{\partial y} = 4y$$

$$\frac{\partial f}{\partial x} = \lambda \frac{\partial g}{\partial x}$$

$$\frac{\partial f}{\partial y} = \lambda \frac{\partial g}{\partial y}$$

$$g(x,y) = 1$$

$$2x = \lambda 2x$$

$$12y^2 = \lambda 4y$$

$$x^2 + 2y^2 = 1$$

$$2x = 2\lambda x$$

$$x(\lambda - 1) = 0$$

$$\downarrow$$

$x = 0$ or $\lambda = 1$

$\lambda = 1 \qquad 12y^2 = 4y \Rightarrow 3y^2 = y \Rightarrow y = 0$ or $y = \frac{1}{3}$

$$x^2 + 2y^2 = 1$$

$$y = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

$$y = \frac{1}{3} \Rightarrow x^2 + \frac{2}{9} = 1 \Rightarrow x = \pm \frac{\sqrt{7}}{3}$$

$$(1,0),\ (-1,0),\ \left(\frac{\sqrt{7}}{3}, \frac{1}{3}\right),\ \left(-\frac{\sqrt{7}}{3}, +\frac{1}{3}\right) \quad \lambda = 1$$

$x = 0 \Rightarrow 0^2 + 2y^2 = 1 \Rightarrow y^2 = \frac{1}{2} \Rightarrow y = \pm \frac{1}{\sqrt{2}}$

$\left(0, \frac{1}{\sqrt{2}}\right) \quad \left(0, -\frac{1}{\sqrt{2}}\right)$