Optimization

min/maxs happen at spots where $\vec{\nabla} f = \vec{0}$.

$$\text{Hess } f = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

x^2+y^2

$-x^2-y^2$

x^2-y^2

$$D = f_{xx} f_{yy} - f_{xy}^2$$

$D < 0$ at a spot where $\vec{\nabla} f = 0 \Rightarrow$ saddle

$D > 0 \Rightarrow$ local min/max

$f_{xx} > 0$ or $f_{yy} > 0 \Rightarrow$ local min

$f_{xx} < 0$ or $f_{yy} < 0 \Rightarrow$ local max

$$f(x,y) = x^2 + 6xy + y^2$$

$$\nabla f = \langle 2x+6y,\ 6x+2y \rangle$$

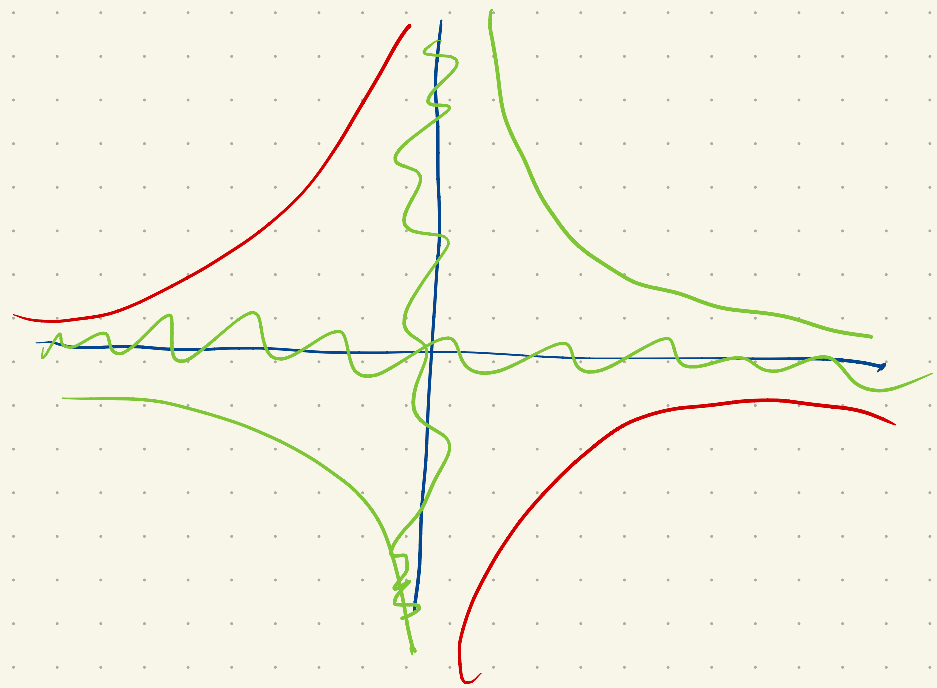
$$2x+6y=0$$
$$6x+2y=0 \Rightarrow \begin{matrix} x=0 \\ y=0 \end{matrix}$$

$$f_{xx}=2 \quad f_{xy}=6$$
$$f_{yx}=6 \quad f_{yy}=2$$

$$\begin{bmatrix} 2 & 6 \\ 6 & 2 \end{bmatrix}$$

$$D<0$$

$$g(x,y)=xy$$



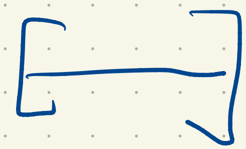
$$f(x,y) = x^2 - y^2$$

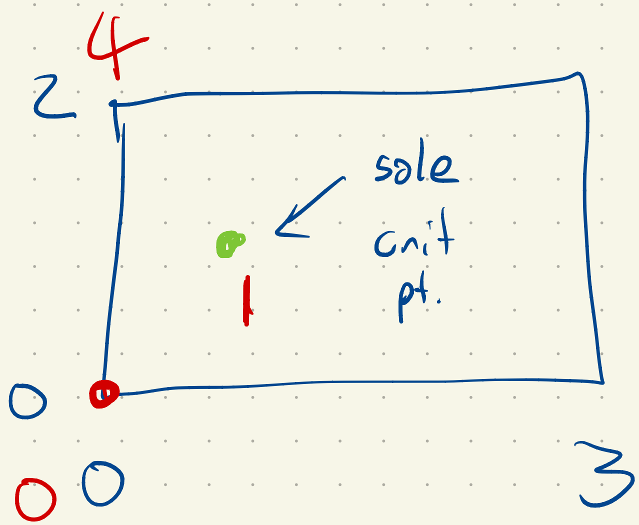
$$\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$f(x,y) = x^2 - 2xy + 2y$$

$0 \le x \le 3$

$0 \le y \le 2$





region: bounded
closed

guaranteed: will be a min and a max value.

$$\vec{\nabla} f = \langle 2x - 2y, -2x + 2 \rangle$$

$$\vec{\nabla} f = 0 \Rightarrow -2x + 2 = 0 \Rightarrow x = 1; \; 2x - 2y = 0 \Rightarrow y = 1$$

$f(1,1) = 1$

Now look at boundary.

$\boxed{x = 0 \qquad 0 \le y \le 2}$

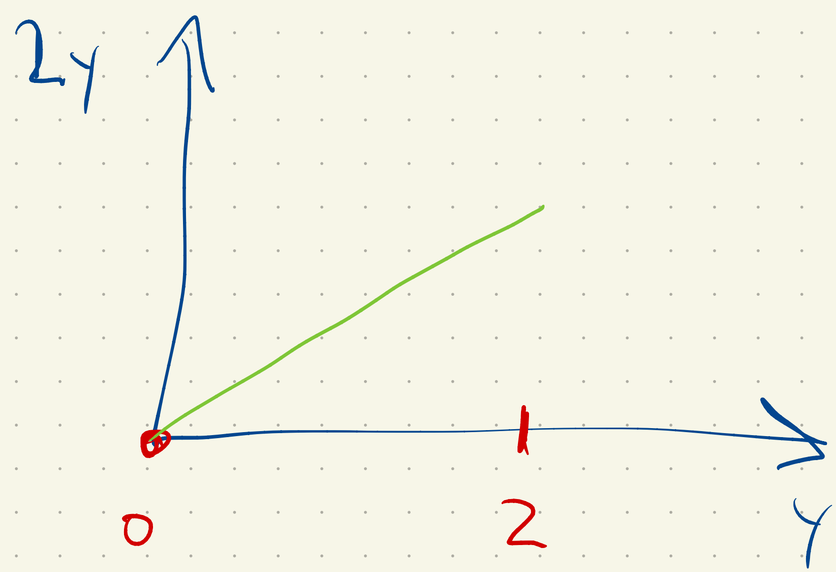
$x = 3 \qquad 0 \le y \le 2$

$f(x,y) = x^2 - 2xy + 2y$

$0 \le x \le 3 \qquad y = 0$

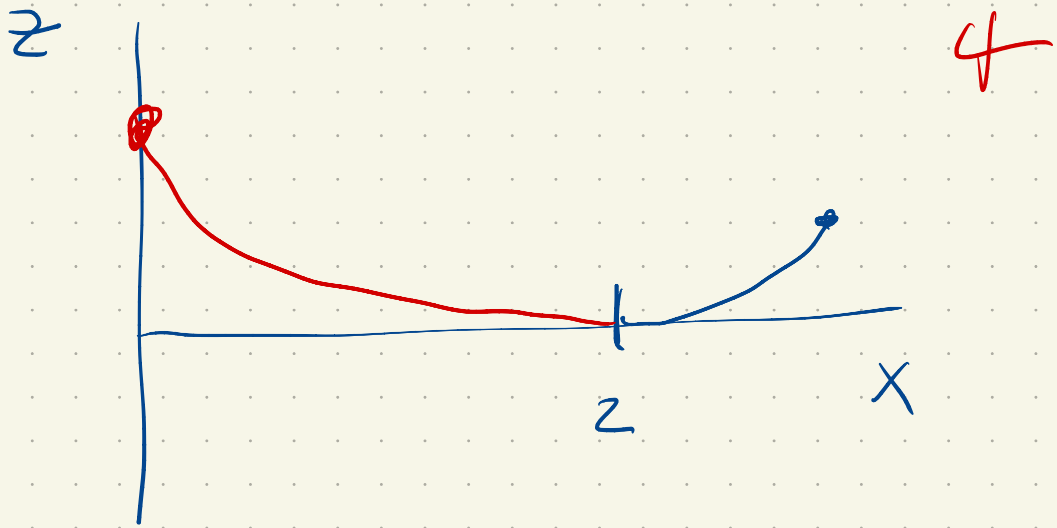
$f(0,y) = 2y$

$0 \le x \le 3 \qquad y = 2$



$y = 2, \quad \underline{0 \le x \le 3}$

$$f(x,y) = x^2 - 4x + 4$$
$$= (x-2)^2$$



$$\langle t^2-4, -t+2t^2\rangle\langle -t, t^2\rangle$$

$$-t^3+4t-t^3+2t^4$$

$$2t^4-2t^3+4t=0$$

$$-t^2+4-2t^2+4t^3$$

$$4t^3-3t^2+4$$