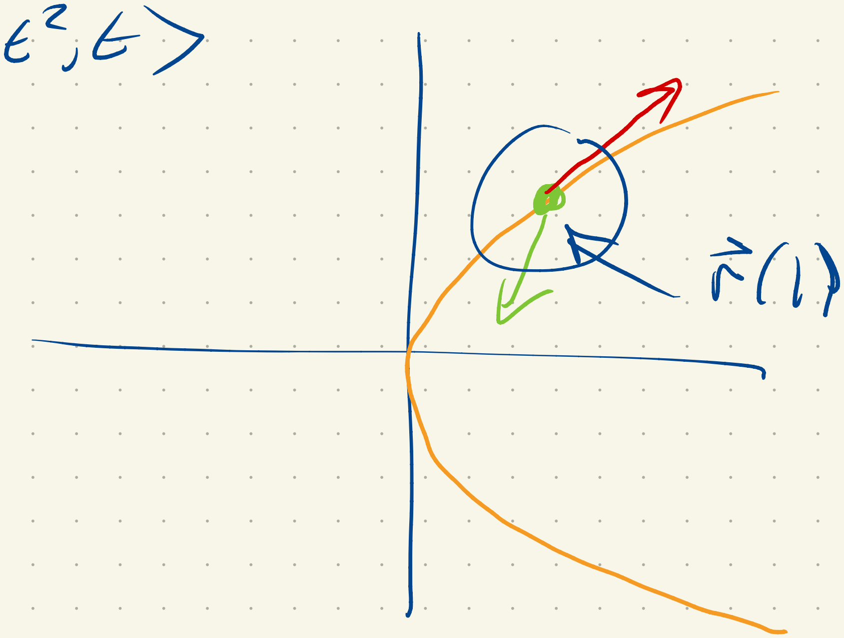
$$\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

Hessian

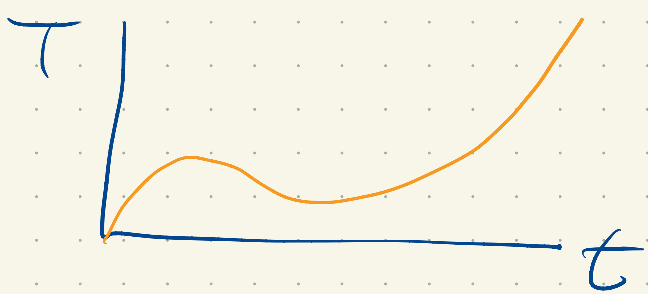
$T(x,y)$

$$\vec{\nabla} T = \langle y-4,\ x+2y \rangle$$

$$\vec{r}(t) = \langle t^2, t \rangle$$



$$\frac{d}{dt} \underbrace{T(\vec{r}(t))} = 0$$



$t=1 \qquad \vec{r}(1) = \langle 1, 1 \rangle$

$\vec{r}\,'(1) = \langle 2, 1 \rangle$

$\vec{\nabla} T(1,1) = \langle -3, 3 \rangle$

$$\frac{d}{dt} T(\vec{r}(t)) = \vec{\nabla} T \cdot \vec{r}\,'$$

at $t=1$

$\langle 2, 1 \rangle \cdot \langle -3, 3 \rangle =$

$-6 + 3 = -3$

$$\boxed{\frac{d}{dt} T(\vec{r}(t)) = \vec{\nabla} T \cdot \vec{r}\,'}$$

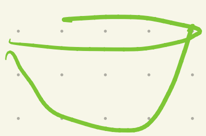
2^{nd} derivative test $f(x,y)$

$$\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$ Hessian

(discriminant)

determant $f_{xx} f_{yy} - (f_{xy})^2 = D$

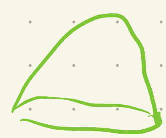
$f(x,y) = x^2 + y^2$



$f_x = 2x$

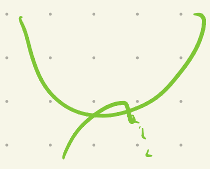
$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$f(x,y) = -x^2 - y^2$



$$\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$f(x,y) = x^2 - y^2$



$$\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

$D = 4$ $\quad$ $D = 4$

either a local min or local max

local min: both diagonal entries pos

local max: both diag entries neg

$D = -4$

saddle

$D = 0$

inconclusive

$$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$$

$$f(x,y) = xy(x-2)(y+3)$$

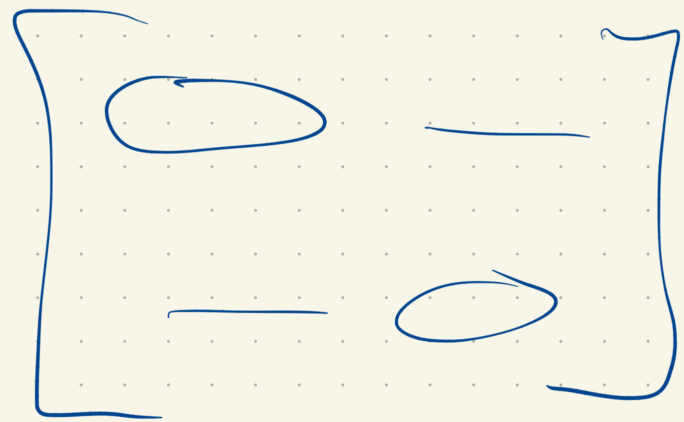
$$f_x = y(x-2)(y+3) + xy(y+3)$$
$$= y(y+3)(2x-2)$$

$$f_y = x(x-2)(y+3) + xy(x-2)$$
$$= x(x-2)(2y+3)$$

$$f_{xx} = \frac{\partial}{\partial x}\left(y(y+3)(2x-2)\right) = y(y+3)\cdot 2$$
$$= 2y(y+3)$$

$$f_{xy} = (y+3)(2x-2) + y(2x-2)$$

$$= (2y+3)(2x-2)$$



$$f_{yy} = 2x(x-2)$$

$$D = f_{xx} f_{xy} - (f_{xy})^2$$

$$= 2y(y+3)2x(x-2) - \left[(2y+3)(2x-2)\right]^2$$

$$= 4xy(y+3)(x-2) - \left[(2y+3)(2x-2)\right]^2$$

$$\left.\begin{matrix} (0,0) & (0,-3) \\ (2,0) & (2,-3) \\ & (1,-\tfrac{3}{2}) \end{matrix}\right] \quad \nabla f = 0$$

$$\begin{bmatrix} f_{xx} & 0 \\ 0 & f_{yy} \end{bmatrix}$$

At $(1,-\frac{3}{2})$

$$D = 4\cdot\left(-\tfrac{3}{2}\right)\left(-\tfrac{3}{2}+3\right)(1-2) - \left((0)\cdot 0\right)^2$$

$$= 4\left(-\tfrac{3}{2}\right)\left(+\tfrac{3}{2}\right)(-1)$$

$$= 9$$

0 0

0

0 0

$(x = 1, y = -\frac{3}{2})$

$$f_{xx} = 2y(y+3) = 2 \cdot \left(-\frac{3}{2}\right)\left(-\frac{3}{2}+3\right)$$

$$= -3 \cdot \left(\frac{3}{2}\right) = -\frac{9}{2} < 0$$

$f_{yy} < 0$

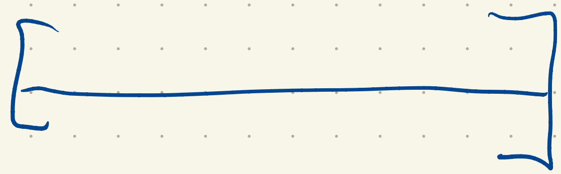
$$\boxed{D > 0 \quad f_{xx} < 0} \longrightarrow \text{local max}$$

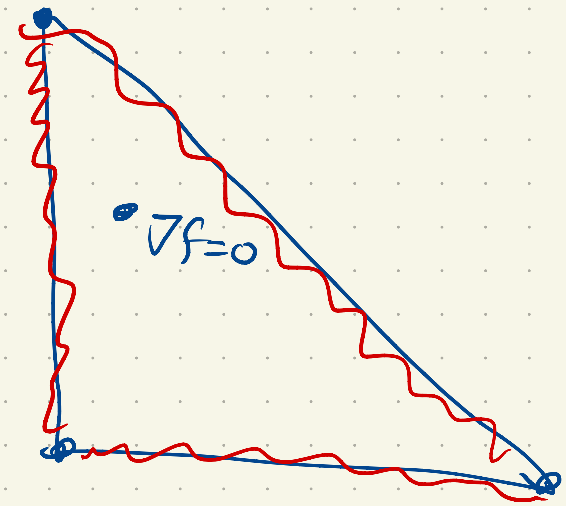
At other 4 points $D < 0$ (saddle)

S S

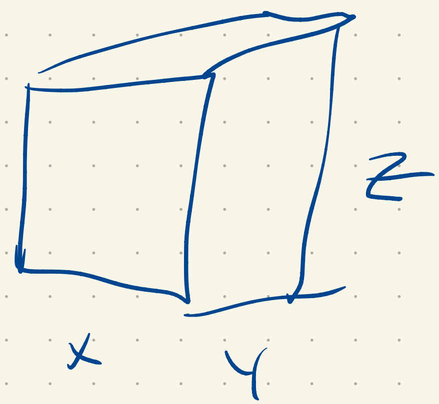
local max

S S





Maximize $V = x \cdot y \cdot z$



$x + y + z \le 96$ inches

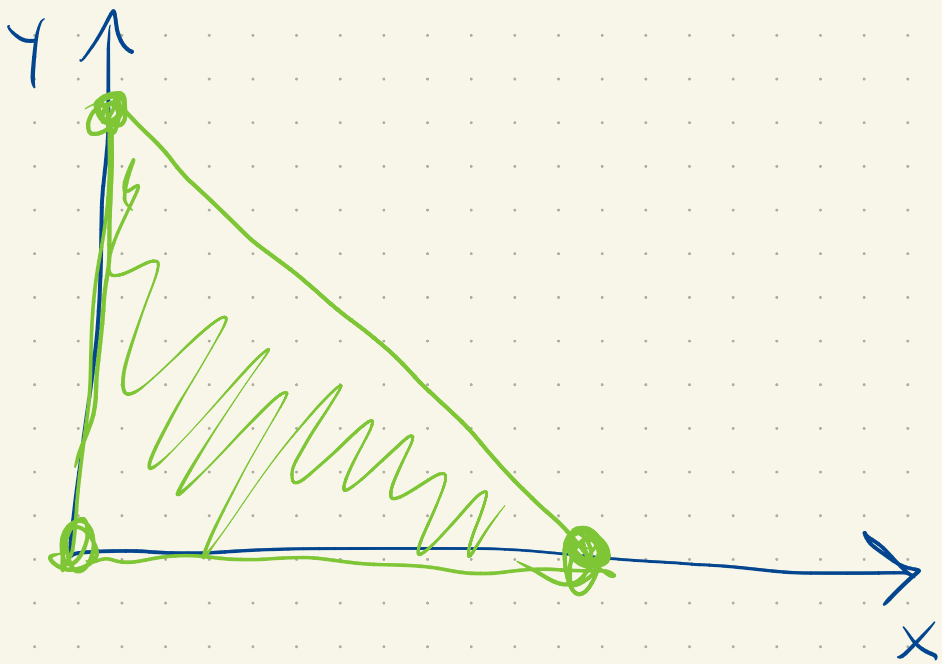
$z = 96 - x - y$

$$V = xy(96 - x - y)$$

$x \geq 0$

$y \geq 0$

$96 - x - y \geq 0$



$96 - x - y = 0$

$y = 96 - x$