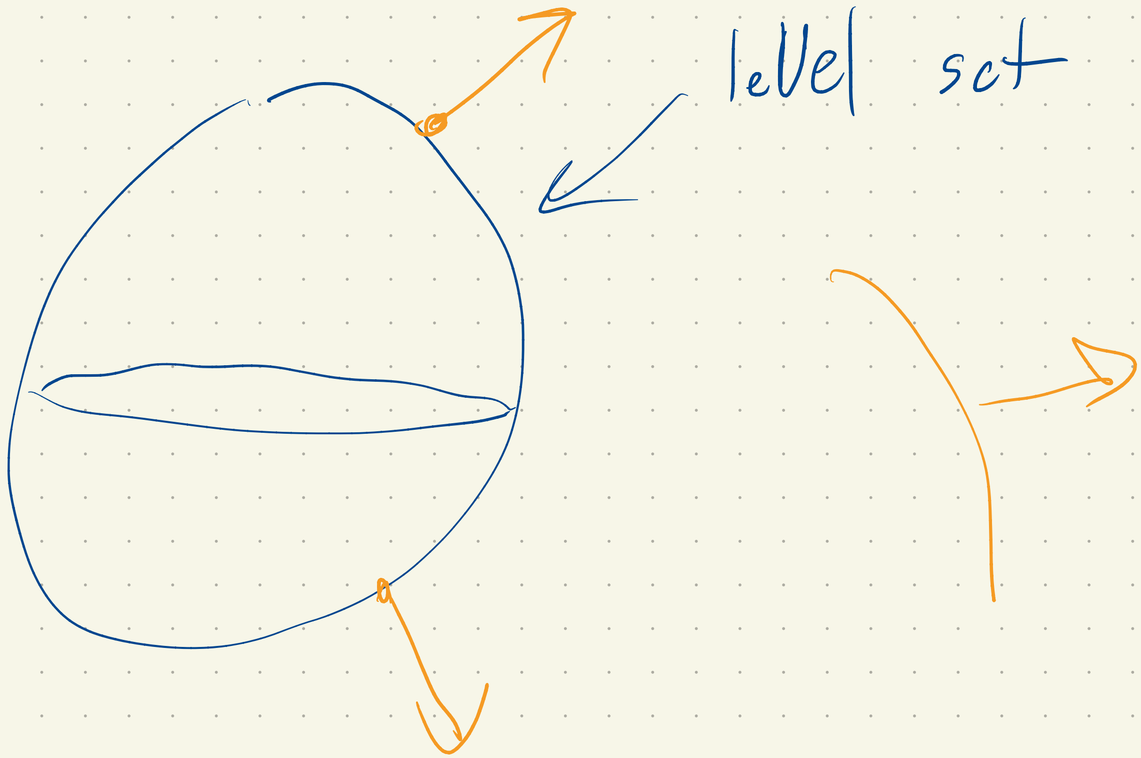
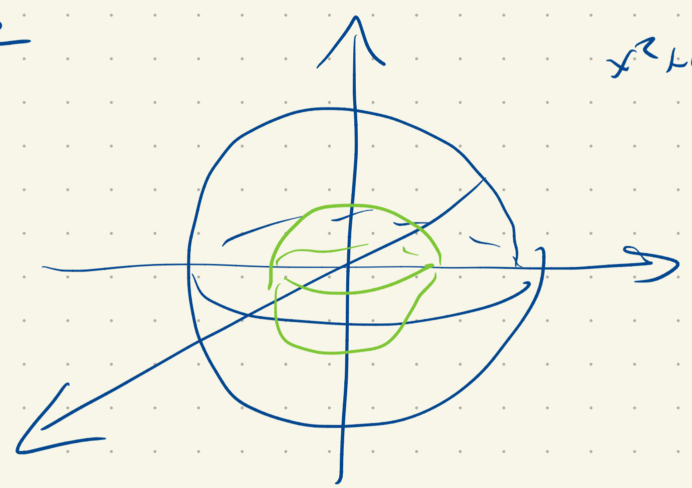


Gradient is perp to level sets of a function

$$f(x,y,z) = x^2 + y^2 + z^2$$

$$f(x,y,z) = 1$$

$$f(x,y,z) = \frac{1}{4}$$



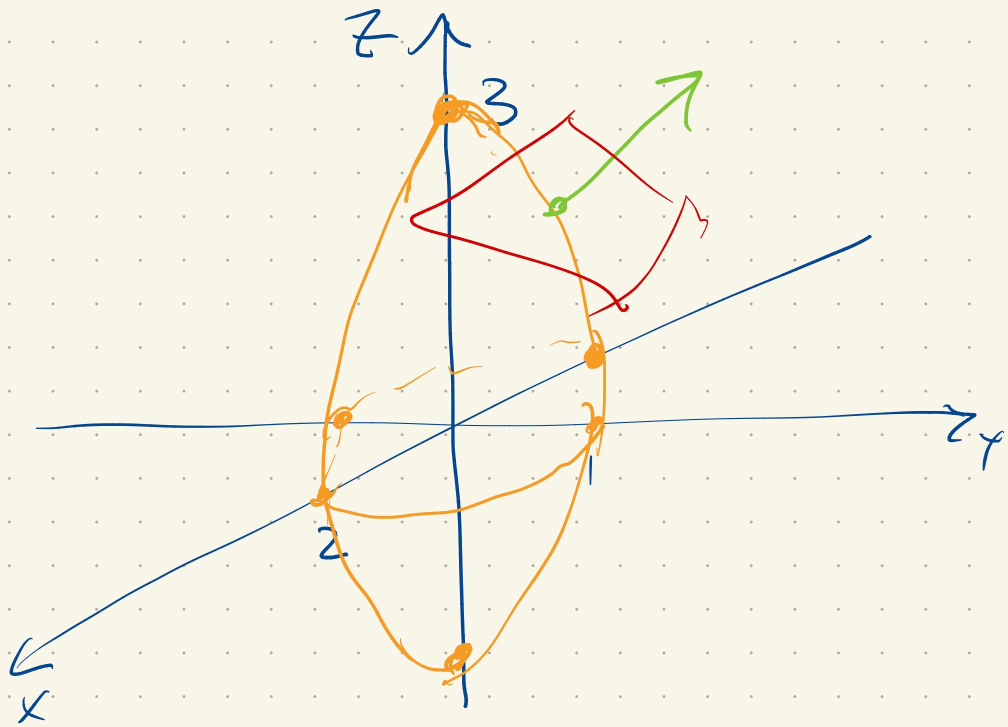
$x^2+y^2+z^2=1$

$x^2+y^2+z^2=\frac{1}{4}$

r^2

$r=\frac{1}{2}$

$$f(x,y,z) = \frac{x^2}{4} + y^2 + \frac{z^2}{9}$$



$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 1$$

$$\left(\frac{x}{2}\right)^2 + y^2 + \left(\frac{z}{3}\right)^2 = 1$$

Task: Find the equation of the tangent plane to the level set of $f(x,y,z) = \left(\frac{x}{2}\right)^2 + y^2 + \left(\frac{z}{3}\right)^2 = \frac{x^2}{4} + y^2 + \frac{z^2}{9}$ at the point $(x,y,z) = (-2, 1, -3)$.

We need: a point on the plane and a normal.

Normal: use gradient!

$$\vec{\nabla} f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle = \left\langle \frac{x}{2}, 2y, \frac{2z}{9} \right\rangle$$

at $(x,y,z) = (-2,1,-3)$

$$\vec{\nabla} f = \left\langle \frac{-2}{2}, 2\cdot 1, \frac{2\cdot(-3)}{9} \right\rangle = \left\langle -1, 2, -\frac{2}{3} \right\rangle$$

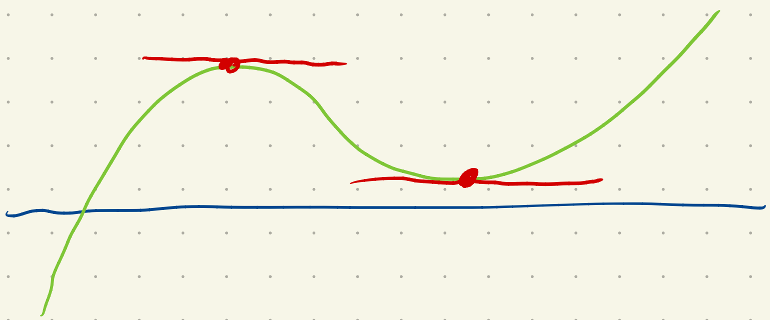
$$-1(x+2) + 2(y-1) - \frac{2}{3}(z+3) = 0$$

$$\vec{n} \cdot \left(\langle x,y,z \rangle - \langle x_0, y_0, z_0 \rangle \right) = 0$$

Section 4.7 Optimization

Facts from calc I

a) If $f(x)$ attains a local max/min at a point x then $f'(x) = 0$.



b) If $f'(x_0) = 0$ and $f''(x_0) > 0$ then local min

$f(x) = x^2$

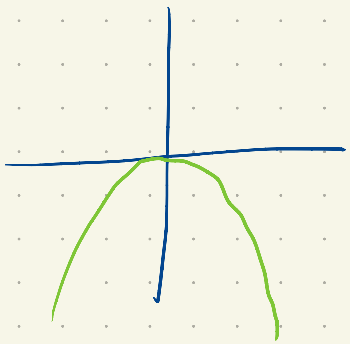
$f'(x) = 2x$

$f''(x) = 2$

$f(x) = -x^2$

$f'(x) = -2x$

$f''(x) = -2$



If $f'(x_0) = 0$ and $f''(x_0) < 0$ then local max

If $f'(x_0) = 0$ and $f''(x_0) = 0$

$f(x) = x^3$

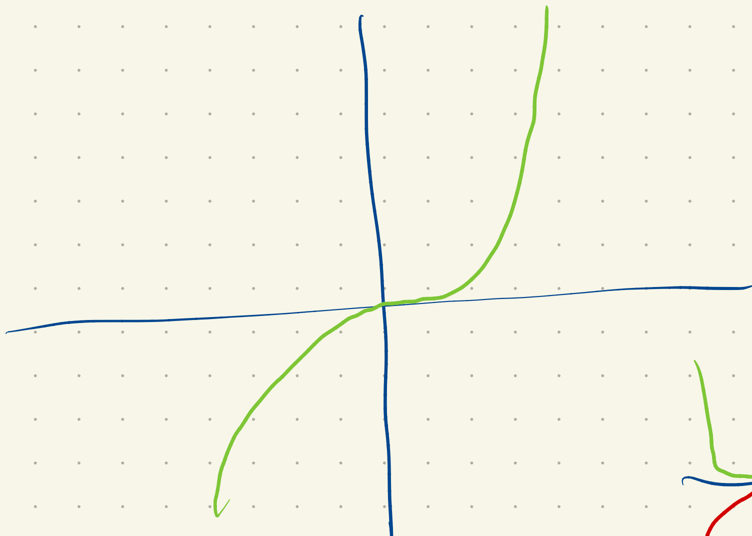
$f'(x) = 3x^2$

$f''(x) = 6x$

$f(0) = 0$

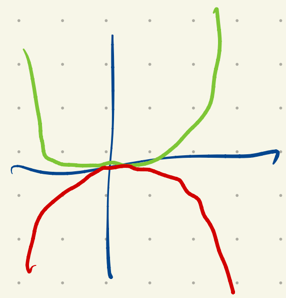
$f'(0) = 0$

$f''(0) = 0$

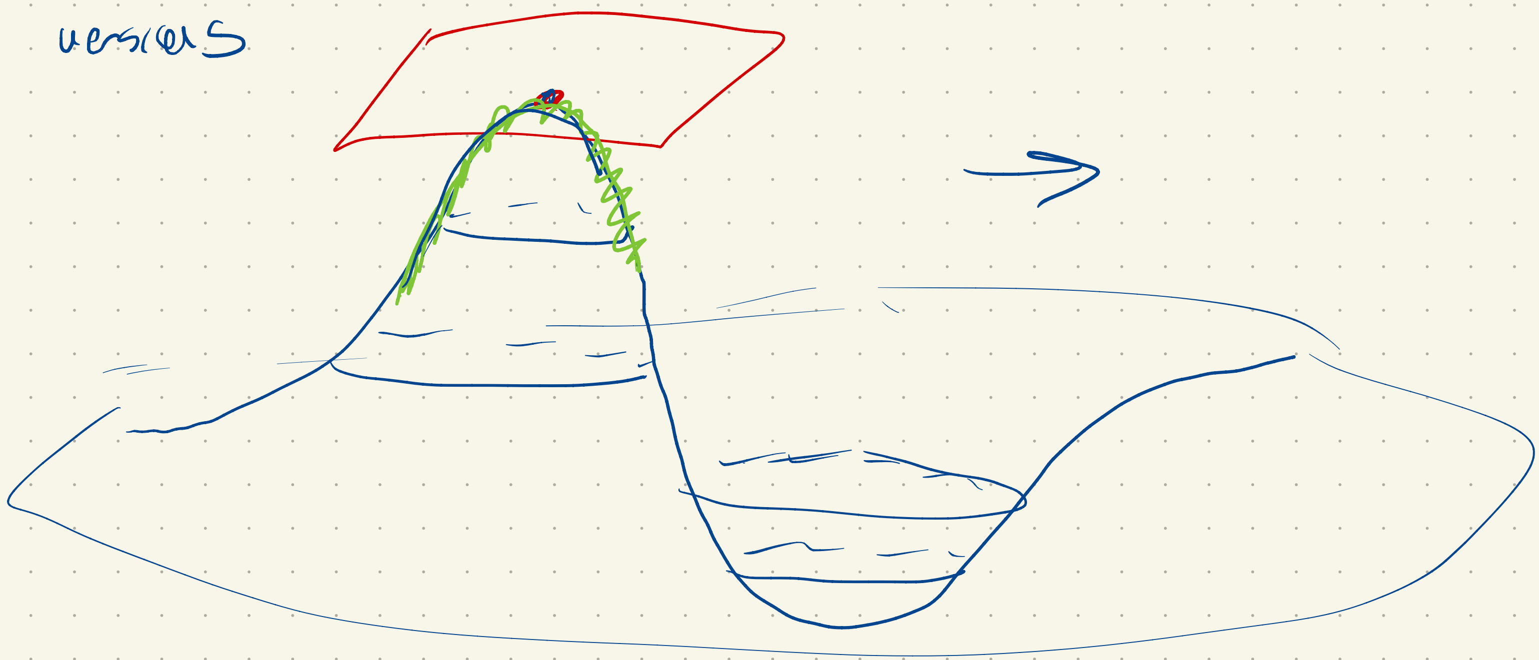


$f(x) = x^4$

$f(x) = -x^4$



New versions



If we have a local max at a point (x_0, y_0)

$$\frac{\partial f}{\partial x}(x_0, y_0) = 0 \qquad \frac{\partial f}{\partial y}(x_0, y_0) = 0 \qquad \frac{\partial f}{\partial z}(\vec{\ }) = 0$$

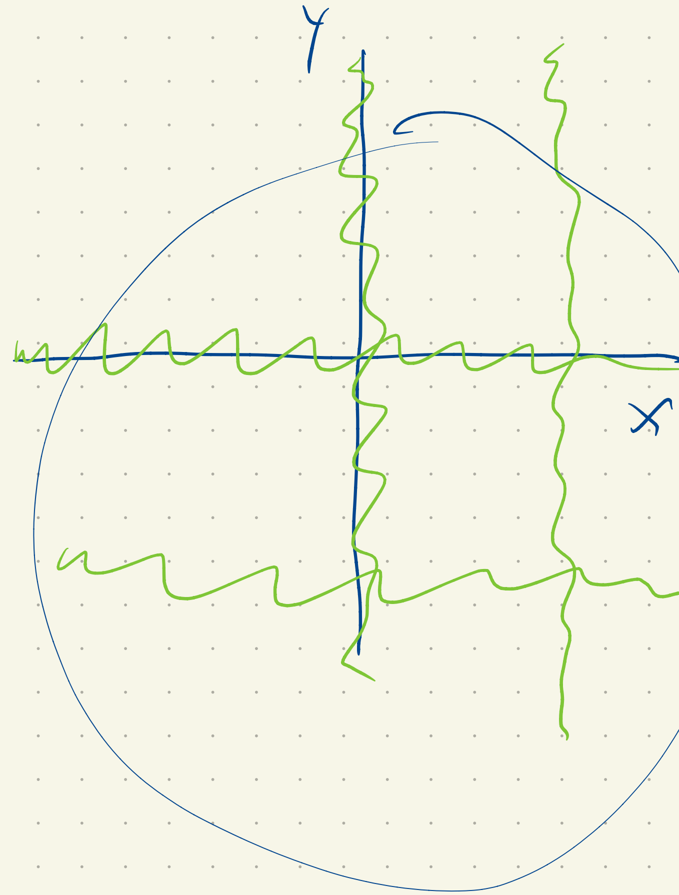
$$\boxed{\vec{\nabla} f(x_0, y_0) = \vec{0}}$$

$$f'(x_0) = 0$$

$$f(x,y) = xy(x-2)(y+3)$$

I want to find local min/max

$$-2y(y+3)(x-1)$$



$$\frac{\partial f}{\partial x} = y(x-2)(y+3) + xy\underbrace{(+1)}_{\uparrow}(y+3)$$

$$= y(y+3)(2x-2)$$

$$= 2y(y+3)(x-1)$$

$$\frac{\partial f}{\partial y} = x(x-2)(y+3) + xy(x-2)$$

$$= x(x-2)(2y+3)$$

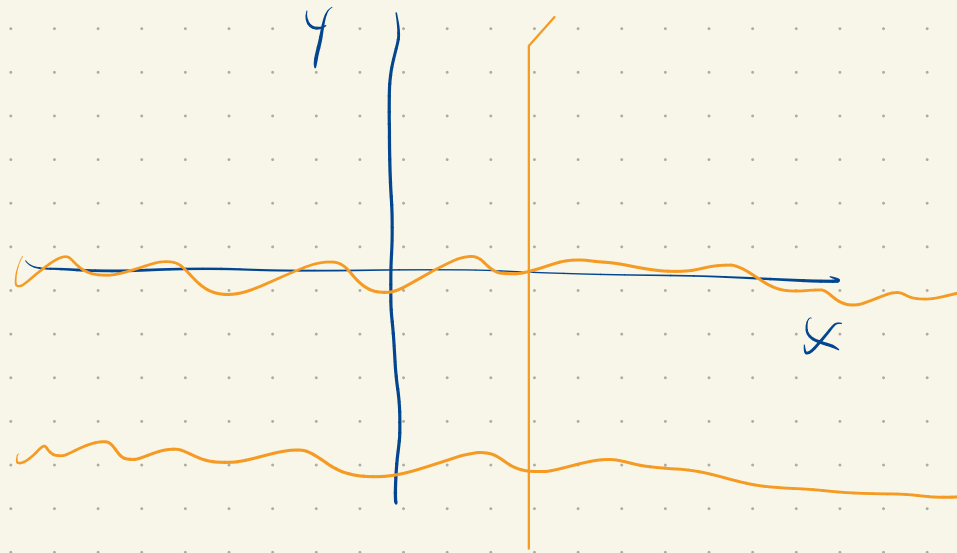
$$\frac{\partial f}{\partial x} = 0 \quad \underline{\text{and}} \quad \frac{\partial f}{\partial y} = 0$$

$$\frac{\partial f}{\partial x} = 2y(y+3)(x-1) \qquad \frac{\partial f}{\partial y} = x(x-2)(2y+3)$$

$\frac{\partial f}{\partial x} = 0$ at $x = 1$

$y = -3$

$y = 0$



$\frac{\partial f}{\partial y} = 0$ at $x = 2$

$x = 0$

$y = -\frac{3}{2}$

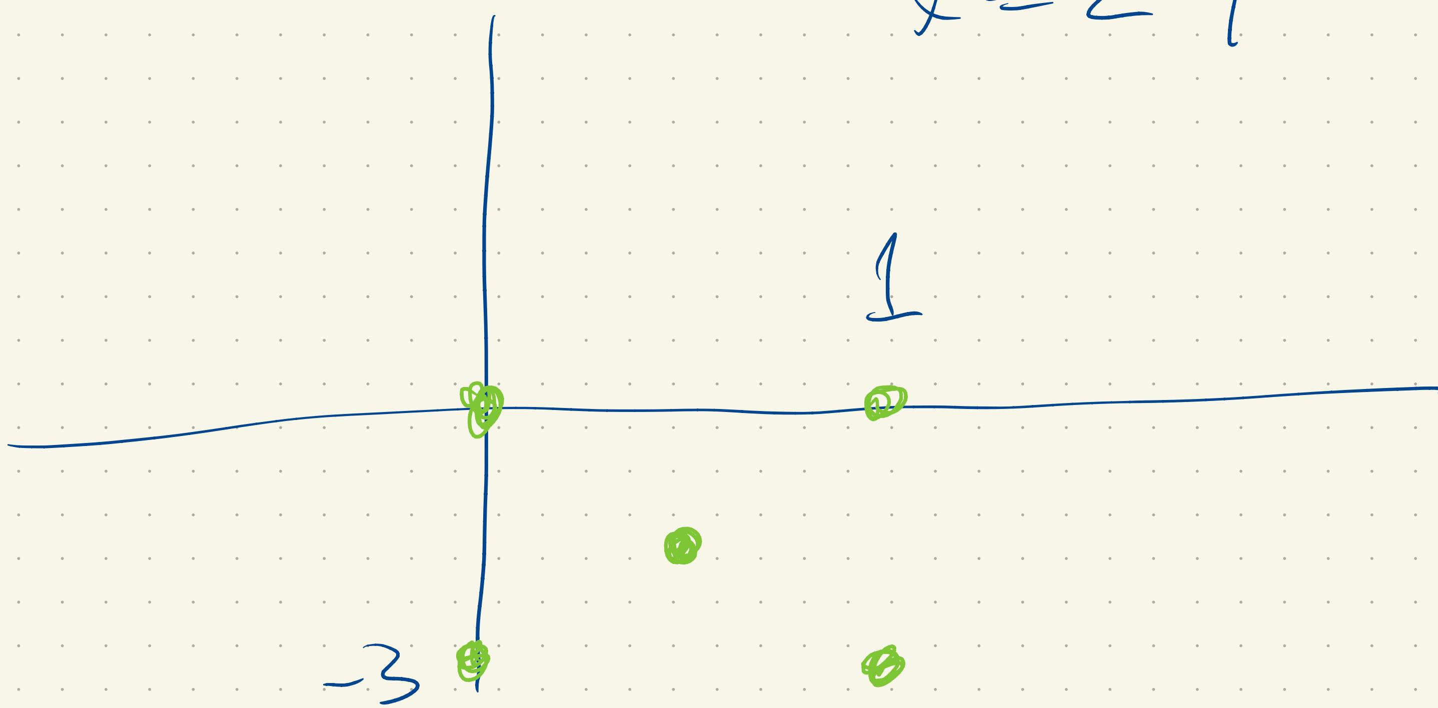
$\vec{\nabla} f = \vec{0}$ at: $x = 1 \quad y = -3/2$

$x = 0 \quad y = -3$

$x = 2 \quad y = -3$

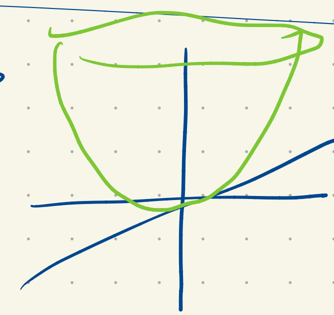
$x = 0 \quad y = 0$

$x = 2 \quad y = 0$

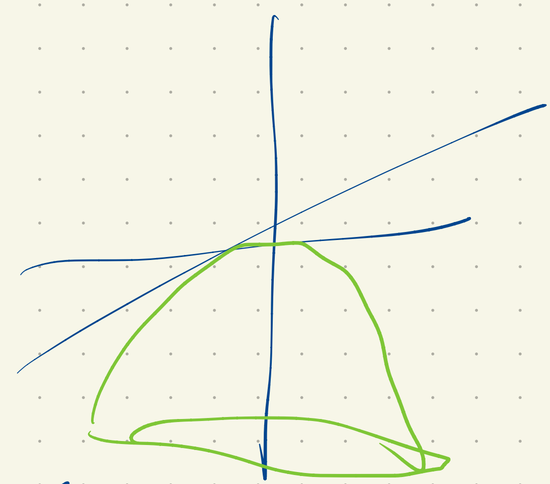


$$f'(x_0) = 0$$

$$f(x,y) = x^2 + y^2$$



$$f(x,y) = -x^2 - y^2$$



$$f(x,y) = x^2 - y^2$$

$$\vec{\nabla} f = \langle 2x, -2y \rangle$$