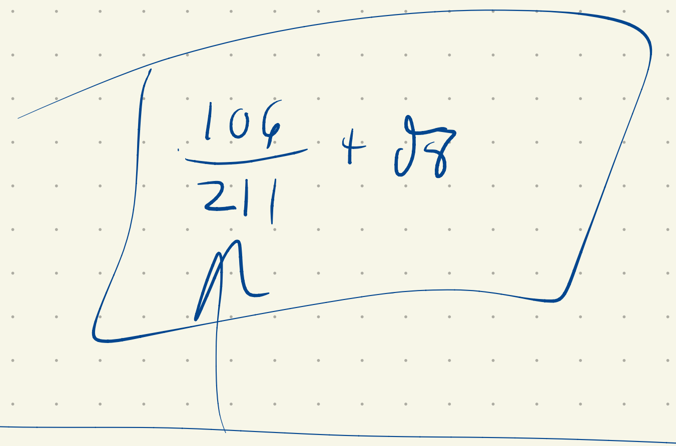
$$z = \frac{1}{2} + \frac{1}{2}(x-2) - \frac{1}{6}(y+1)$$

(tangent plane at $x = 2,\ y = -1$)

$$z = -\frac{2}{3} + \frac{1}{2}x - \frac{1}{6}y$$




$$f(x,y) \qquad \vec{\nabla} f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

1) Its a vector field (one vector at each x,y location)
2) It points in the direction of steepest increase.
3) It is perpendicular to level sets of f.

4) Its length encodes steepness: the longer the gradient the steeper the graph of f is.

5) Most important.

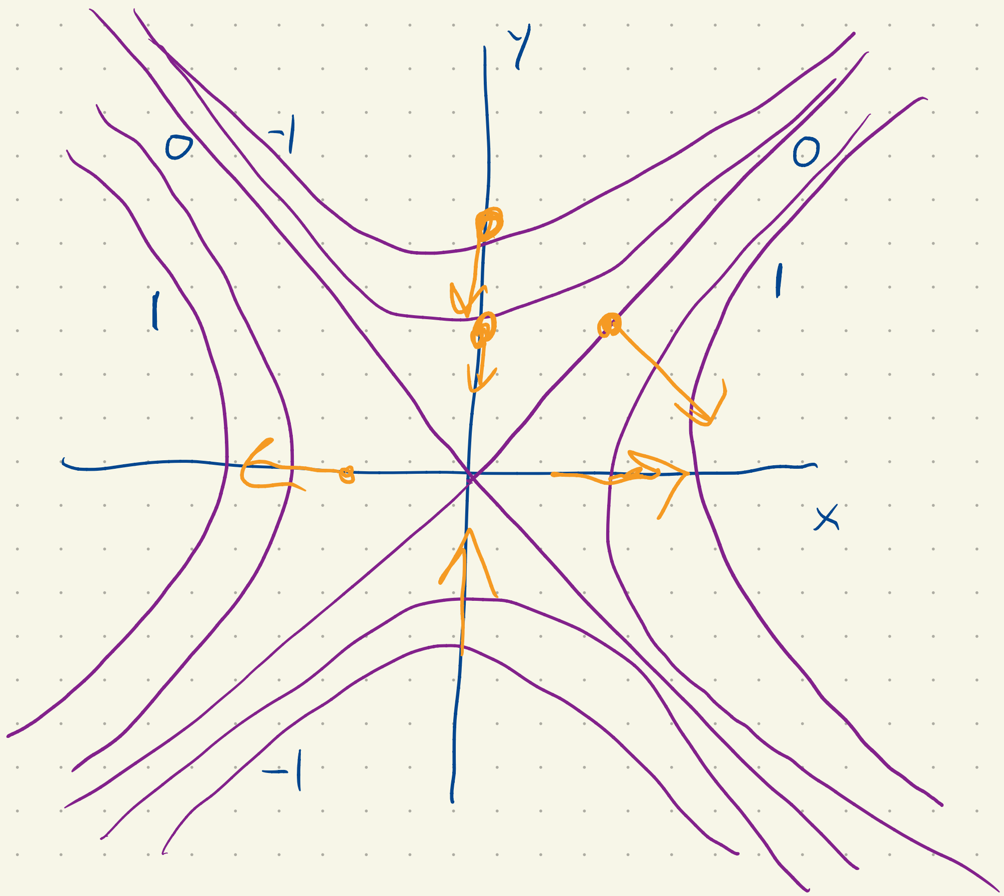
For a curve $\vec{r}(t) = \langle x(t), y(t) \rangle$

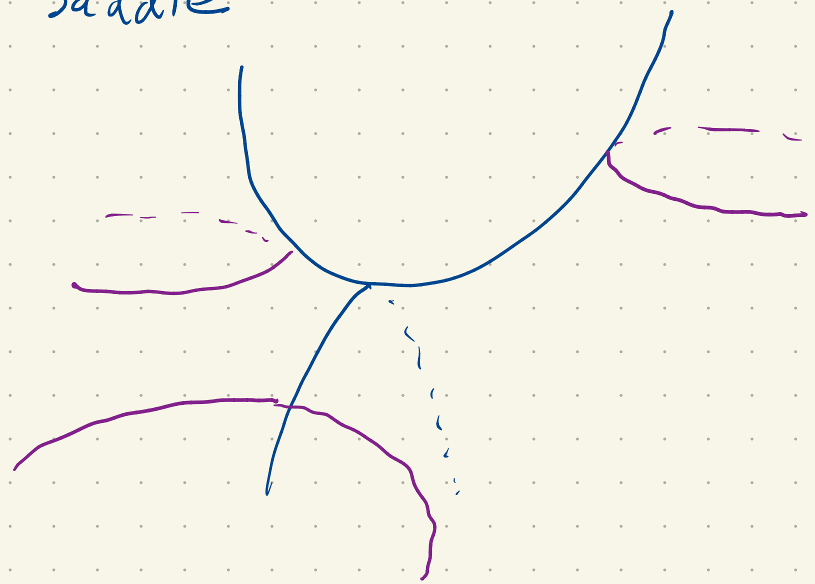
the rate of change of f seen along $\vec{r}$

is $\vec{\nabla} f \cdot \vec{r}\,'$

$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$$

$h(x,y) = x^2 - y^2$ saddle



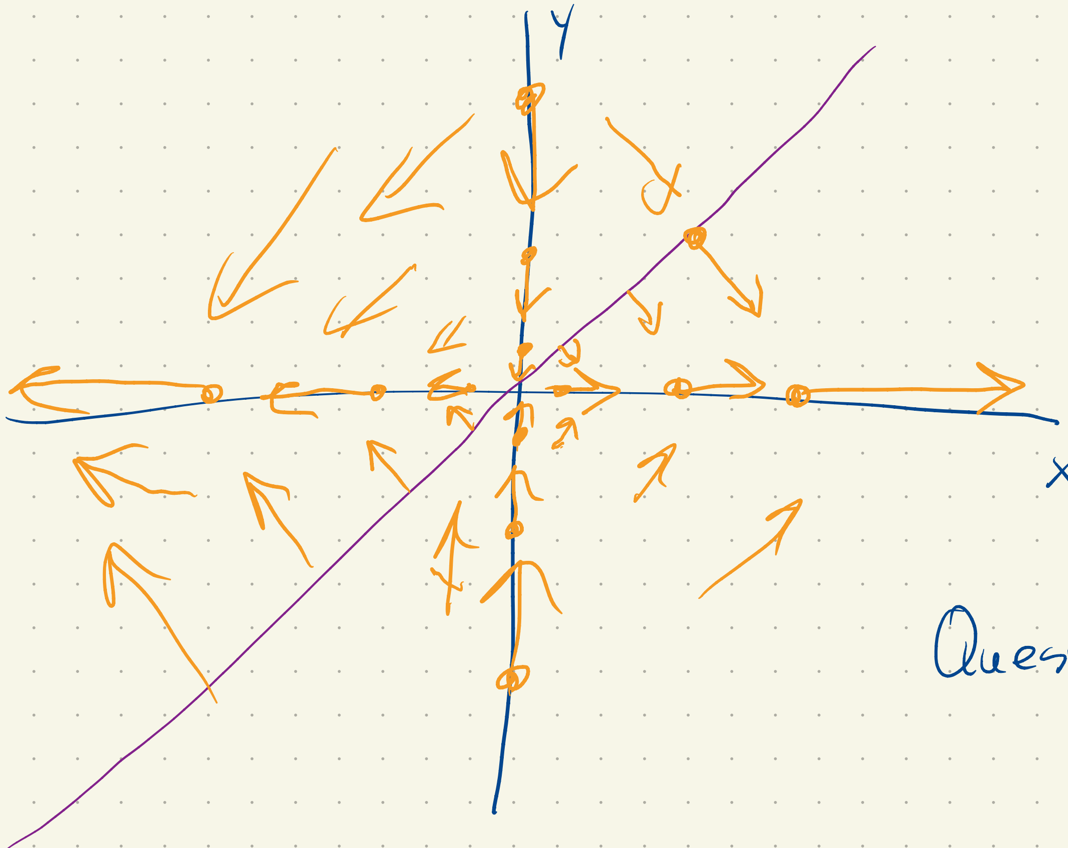
$h(x,y) = x^2 - y^2$

$\vec{\nabla} h = \langle 2x, -2y \rangle$

If $x = 0$ $\quad \vec{\nabla} h = \langle 0, -y \rangle$

If $y = x$ $\quad \vec{\nabla} h = \langle 2x, -2x \rangle$

$= 2x \langle 1, -1 \rangle$




Question: What is the rate of change of $h(x,y)$ at $x = 3$, $y = -1$ if traveling with velocity $\langle 1, -2 \rangle$

$$\vec{\nabla} h = \langle 2x, -2y \rangle$$

at $x=3$ $y=-1$, $\vec{\nabla} h = \langle 6, 2 \rangle$

$$\vec{v} = \langle 1, -2 \rangle$$

rate of change is $\vec{\nabla} h \cdot \vec{v} = 6 \cdot 1 - 2 \cdot 2 = 2$

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Some justification of properties of the gradient.

$$\vec{\nabla} f \cdot \vec{v} = \|\vec{\nabla} f\| \, \|\vec{v}\| \cos\theta$$

Let take $\|\vec{v}\| = 1$. When is $\vec{\nabla} f \cdot \vec{v}$ at a maximum?

$\|\vec{\nabla} f\| \cos\theta$ $\qquad$ $\theta = 0$

($\vec{v}$ points in direction of $\vec{\nabla} f$)

$\vec{\nabla} f \cdot \vec{v} = 0$ when $\theta = \frac{\pi}{2}$, the direction perpendicular to $\vec{\nabla} f$.

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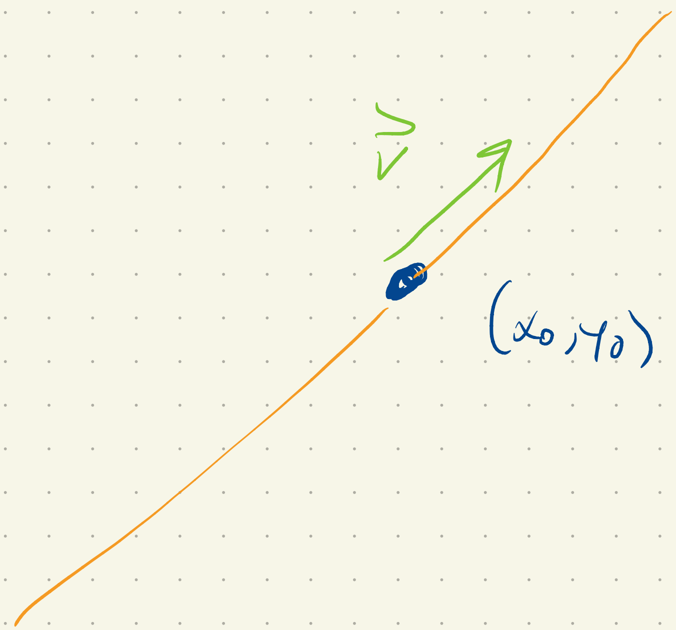
# Directional derivative

$f(x,y)$ $\quad$ $(x_0, y_0)$ point of interest

$\vec{v} = \langle v_x, v_y \rangle$




$\vec{r}(t) = \langle x_0, y_0 \rangle + t \langle v_x, v_y \rangle$

$$D_{\vec{v}} f(x_0, y_0) = \frac{d}{dt}\Big|_{t=0} f(\vec{r}(t))$$

$$= \frac{d}{dt}\Big|_{t=0} f(x_0 + t v_x, y_0 + t v_y)$$

(If I'm starting at $\langle x_0, y_0 \rangle$ and travelling with velocity $\vec{v}$

then $D_{\vec{v}} f$ is exactly the the rate of change of $f$

that I am seeing).

For most functions $D_{\vec{v}} f = \vec{\nabla} f \cdot \vec{v}$

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

This function has directional derivatives in every direction at the origin but it doesn't allow a good tangent plane approximation. The gradient fails to capture all the directional derivatives

If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist near $(x_0, y_0)$ and are continuous then the tangent plane approximation at $(x_0, y_0)$ is "good" and every directional derivative can be computed at $(x_0, y_0)$ by $\vec{\nabla} f \cdot \vec{v} = D_{\vec{v}} f$.

unit vector

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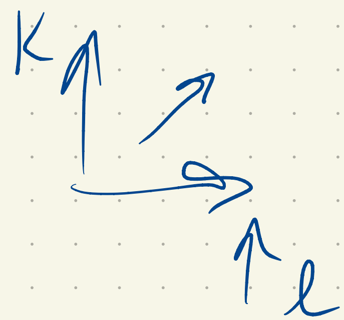
$$P = 8.2 \frac{T}{V}$$

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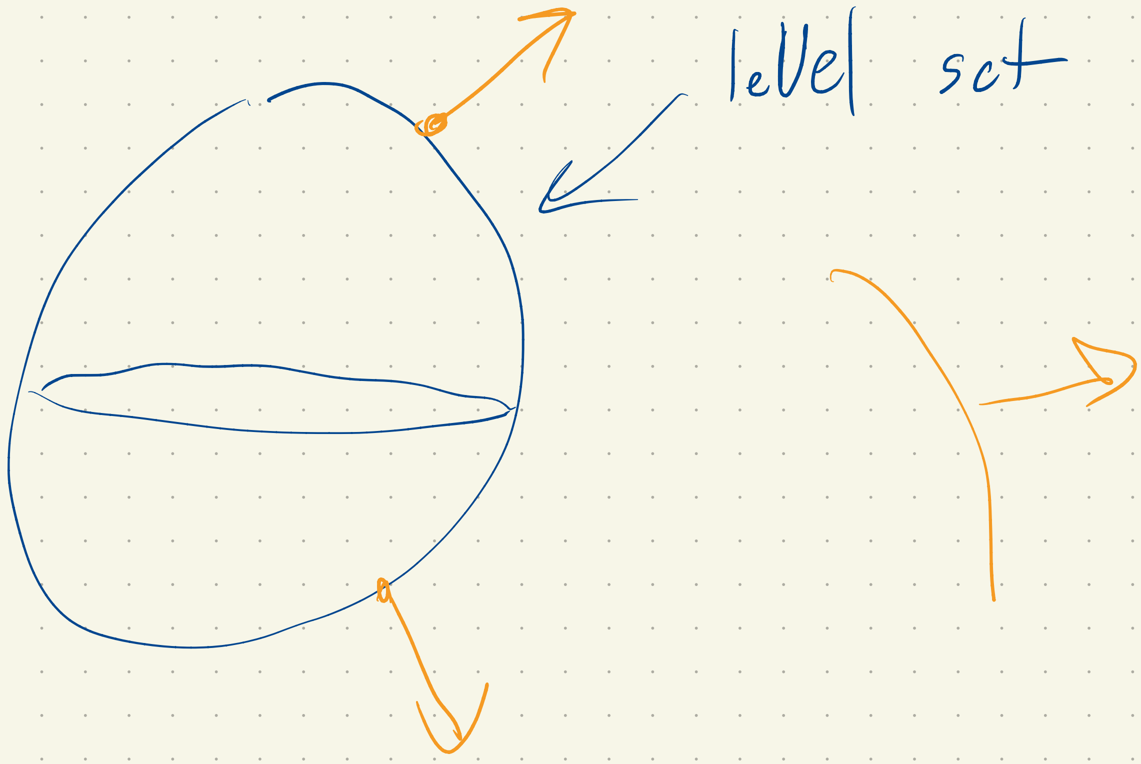
$$f(x,y,z) \qquad \vec{\nabla} f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$


level set