Gradient

$$T(x,y) \qquad \vec{r}(t) = \langle x(t), y(t) \rangle$$

$$\boxed{\frac{d}{dt}\, T(\vec{r}(t)) = \frac{\partial T}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial T}{\partial y} \cdot \frac{dy}{dt}}$$

$$\vec{r}\,'(t) = \langle x'(t), y'(t) \rangle = \left\langle \frac{dx}{dt}(t), \frac{dy}{dt}(t) \right\rangle$$

$$= \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$$

$$\vec{w} = \langle a, b \rangle$$

$$\vec{w} \cdot \vec{r}\,' = a\frac{dx}{dt} + b\frac{dy}{dt}$$

$$\boxed{\left\langle \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y} \right\rangle \cdot \vec{r}\,' = \frac{\partial T}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial T}{\partial y} \cdot \frac{dy}{dt}}$$

The vector $\left\langle \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y} \right\rangle$ is called the gradient at T

and is written $\vec{\nabla} T$.

The vector $\vec{\nabla} T$ depends on (x, y).

$$T(x,y) = x^2 + y^2 \qquad \vec{\nabla} T = \left\langle \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y} \right\rangle$$

$$= \langle 2x, 2y \rangle$$

At the origin $x=0, y=0$ $\quad \vec{\nabla} T = \langle 0, 0 \rangle$

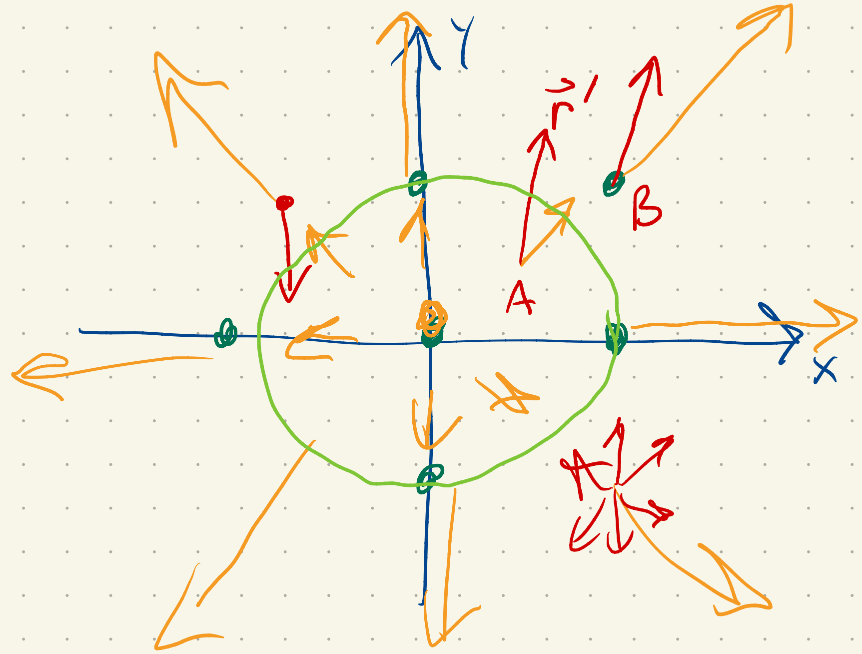
At $x=0, y=1$ $\quad \vec{\nabla} T = \langle 0, 2 \rangle$

$x=1, y=0$ $\quad \vec{\nabla} T = \langle 2, 0 \rangle$

$x=0, y=-1$ $\quad \vec{\nabla} T = \langle 0, -2 \rangle$

$x=-1, y=0$ $\quad \vec{\nabla} T = \langle -2, 0 \rangle$

$x=1, y=1$ $\quad \vec{\nabla} = \langle 2, 2 \rangle$

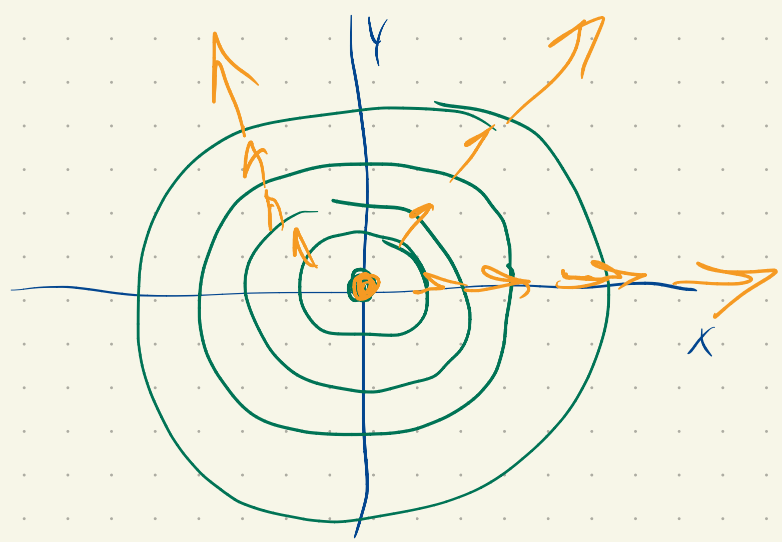


gradient vector field

$$\frac{d}{dt} T(\vec{r}(t)) = \vec{\nabla} T \cdot \vec{r}\,'$$

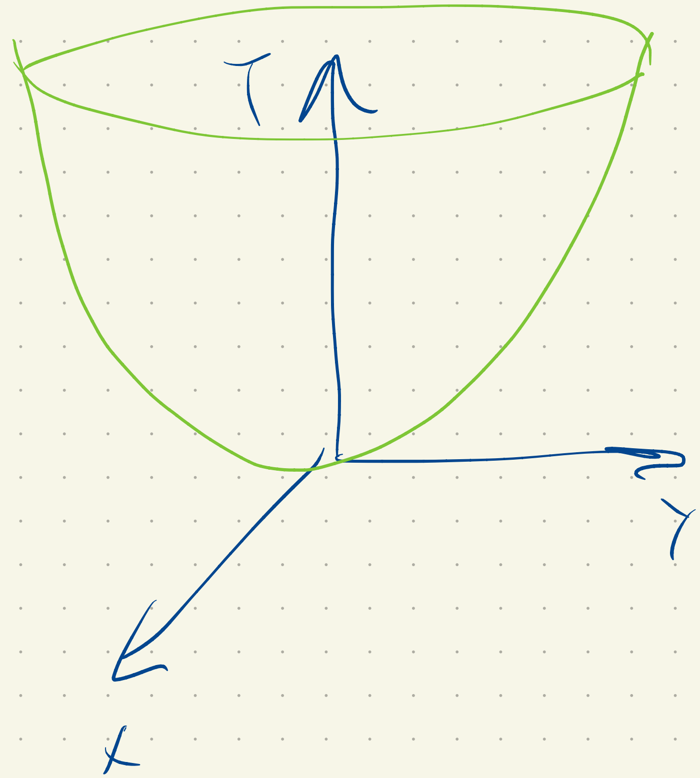
$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \, \|\vec{b}\| \cos\theta \qquad \vec{\nabla} T \cdot \vec{r}\,' = \|\vec{\nabla} T\| \, \|\vec{r}\,'\| \cos\theta$$

The gradient is always perpendicular to level sets (!)



$$T(x,y) = x^2 + y^2$$

$$x^2 + y^2 = 1$$



T
Y
X